Name:

Due Wed Jan. 20 in class.
Problems with $(*)$ are required but no need to hand in.

1. $(*)$ Read lecture notes Sections 1.1, or corresponding materials in textbook.

2. Provide an example of a measure space $(\Omega, \mathcal{F}, \mu)$ with a family of measurable sets $\{A_n\}_{n\in\mathbb{N}} \subset \mathcal{F}$ such that $A_n \downarrow A \in \mathcal{F}$ as $n \to \infty$ but $\lim_{n\to\infty} \mu(A_n) \neq \mu(A)$.

3. Let $\Omega = \mathbb{N}$, and $\mathcal{F}$ be the collection of all subsets A of $\mathbb{N}$ such that either A or A^c is finite. For all $A \in \mathcal{F}$, set $P(A) = 0$ if A is finite, and $P(A) = 1$ if A^c is finite.

   (i) Show that $\mathcal{F}$ is *not* a σ-algebra.

   (ii) Does there exist a probability space $(\mathbb{N}, \mathcal{G}, \mathbb{P})$ such that: $\mathcal{F} \subset \mathcal{G}$, and $\mathbb{P}(A) = P(A)$ for all $A \in \mathcal{F}$? Justify your answer.

4. Consider $\Omega = \{a, b, c, d\}$. Find a strictly increasing sequence of σ-algebras $\mathcal{F}_1 \subsetneq \mathcal{F}_2 \subsetneq \cdots \subsetneq \mathcal{F}_n$ ($\mathcal{A} \subsetneq \mathcal{B}$ means $\mathcal{A} \subset \mathcal{B}$ and $\mathcal{A} \neq \mathcal{B}$) of Ω. What is the largest n that you can get?

5. Prove that in $\mathbb{R}^d$ for arbitrary $d \in \mathbb{N}$, $\sigma(\{(\mathbf{a}, \mathbf{b}] : \mathbf{a} < \mathbf{b}\}) = \sigma(\{[\mathbf{a}, \mathbf{b}] : \mathbf{a} < \mathbf{b}\})$. Provide all details.