Due Monday April 20 in class.
Problems with $(*)$ are required but no need to hand in.

Provide your solutions in a simple and clear manner. In particular, you should be able to read them out aloud without any modification.

1. $(*)$ Read Chapter 3.2, 3.3, 3.4.

2. Exercises 3.2.2 (i), 3.2.12, 3.2.13, 3.2.14 $(*)$, 3.3.5 $(*)$, 3.4.11 $(*)$, 3.4.12.

3. Provide an example of random variables $\{X_n\}_{n\in\mathbb{N}}, X$ and a continuous function g, such that $X_n \Rightarrow X$ as $n \to \infty$, but $\mathbb{E}g(X_n)$ does not converge to $\mathbb{E}g(X)$ as $n \to \infty$.

   Hint: In particular, you can pick $g(x) = x$. The example then will indicate that weak convergence does not imply convergence of the mean.

4. Let $\{X_n\}_{n\in\mathbb{N}}$ be i.i.d. random variables with $\mathbb{E}X_i = \mu, \mathrm{Var}(X_i) = \sigma^2 \in (0, \infty)$. Consider $Y_n = X_n + X_{n+1}, n \in \mathbb{N}$. Show that the central limit theorem holds for $\{Y_n\}_{n\in\mathbb{N}}$:

$$\frac{Y_1 + \cdots + Y_n - n\mu_Y}{\sqrt{n}} \Rightarrow \mathcal{N}(0, \sigma_Y^2),$$

   and identify μ_Y and σ_Y^2.

5. $(*)$ Work out the following prelim problems to be discussed in classroom:

   2014 April 5, 6, 7, 8 (before Monday April 20)

   2013 August 5, 6, 8 (before Wednesday April 22)

   `http://www.artsci.uc.edu/departments/math/grad/PhD_Preliminary_Exams_Semesters/probability-statistics.html`.