Due Wed April 1 in class.
Problems with $(*)$ are required but no need to hand in.

Provide your solutions in a simple and clear manner. In particular, you should be able to read them out aloud without any modification.

1. $(*)$ Read Chapter 2.5, 2.5.1.

2. Exercises 2.3.18 (ii), 2.3.19 $(*)$, 2.5.1, 2.5.4, 2.5.6.

3. Redo Exercise 2.3.15 (iii). Hint: the following inequalities are useful to control $\mathbb{P}(\cup_{i=1}^n A_i)$ from both above and below:

$$\sum_{i=1}^n \mathbb{P}(A_i) - \sum_{1\le i,j\le n, i\neq j} \mathbb{P}(A_i \cap A_j) \le \mathbb{P}\left(\bigcup_{i=1}^n A_i\right) \le \sum_{i=1}^n \mathbb{P}(A_i).$$

   In particular, $\mathbb{P}(\cup_{i=1}^n A_i)$ and $\sum_{i=1}^n \mathbb{P}(A_i)$ are asymptotically equivalent, if $\sum_{i\neq j} \mathbb{P}(A_i \cap A_j) = o(\sum_{i=1}^n \mathbb{P}(A_i))$.

   $(*)$ Prove the above inequalities.

4. Let X and $\{X_n\}_{n\in\mathbb{N}}$ be real-valued random variables defined in a common probability space, such that $X_n \to X$ as $n \to \infty$ in probability, and X_n is non-increasing almost surely. Show that $\lim_{n\to\infty} X_n = X$ almost surely.