Due Friday Feb. 20 in class.
Problems with (∗) are required but no need to hand in.

1. (∗) Read Chapter 1.6, 1.7.

2. Exercises 1.6.3, 1.6.4, 1.6.6, 1.6.11, 1.6.13, 1.6.14.

3. (∗) Exercises 1.5.7, 1.5.8, 1.6.7, 1.6.9, 1.6.10, 1.6.16.

4. Following Exercise 1.6.13, provide an example of $\{X_n\}_{n\in\mathbb{N}}$ such that $\mathbb{E}X_1^- = \infty$, $X_n \uparrow X$ with probability one, but $\mathbb{E}X_n \uparrow \mathbb{E}X$ does not hold.

5. Let $\{b_{m,n}\}_{m,n\in\mathbb{N}}$ be a collection of non-negative real numbers. Consider
$$S_n := \sum_{m\in\mathbb{N}} b_{m,n}, n \in \mathbb{N},$$
and the limit as $n \to \infty$. Assume that for each m, $\lim_{n\to\infty} b_{m,n} = \beta_m \in \mathbb{R}$ and $\sum_{m=1}^{\infty} \beta_m < \infty$.

   (i) Formulate this problem in the language of Fatou's lemma as in textbook, by specifying the measure space $(\Omega, \mathcal{F}, \mu)$ and measurable functions f_n. What does Fatou's lemma tell in this case?

   (ii) By providing a counterexample, show that one should not expect $\lim_{n\to\infty} S_n$ to exist as a finite number, without further assumptions on $b_{m,n}$.

   (iii) Provide an additional assumption on $b_{m,n}$ that guarantees
$$\lim_{n\to\infty} S_n = \sum_{m=1}^{\infty} \beta_m.$$