Name:

Due Mon Feb. 2 in class.
Problems with $(*)$ are required but no need to hand in.

1. $(*)$ Read Chapter 1.2, 1.3.

2. Exercises 1.2.1, 1.2.4 (*), 1.3.1, 1.3.2, 1.3.3.

3. For each of the following distributions, give the explicit construction used in Theorem 1.2.2. You can provide a detailed sketch of the function X as the solution.

   (a) $\mathbb{P}(X = i) = p_i, i = 1, \ldots, n$ with $p_i \geq 0, \sum_{i=1}^n p_i = 1$.

   (b) $(*)$ X has standard exponential distribution $\mathbb{P}(X > x) = e^{-x}, x \geq 0$.

4. Construct two different random variables in the same probability space, so that they have the same distribution. Namely, provide explicitly a probability space $(\Omega, \mathcal{F}, \mathbb{P})$ and two measurable functions $X : (\Omega, \mathcal{F}) \to (\mathbb{R}, \mathcal{B}(\mathbb{R}))$ and $Y : (\Omega, \mathcal{F}) \to (\mathbb{R}, \mathcal{B}(\mathbb{R}))$, such that the two random variables have the same induced distribution, and $\mathbb{P}(\omega : X(\omega) \neq Y(\omega)) > 0$. In particular, provide a construction so that

   (i) both X and Y have Bernoulli (p) distribution. That is, $\mathbb{P}(X = 1) = p = 1 - \mathbb{P}(X = 0)$.

   (ii) both X and Y are uniform $(0, 1)$ distribution. That is, $\mathbb{P}(X \leq x) = x, x \in [0, 1]$.

5. Consider a collection of closed intervals $\{[a_n, b_n]\}_{n \in \mathbb{N}}$. Assume that they are *nested*, that is $[a_n, b_n] \supset [a_{n+1}, b_{n+1}], n \in \mathbb{N}$. Then,

$$\bigcap_{n=1}^{\infty} [a_n, b_n] \neq \emptyset.$$

   We do not prove this result in this class.

   Is the statement remains valid, if we replace all closed intervals $[a_n, b_n]$ above by open intervals (a_n, b_n)? or by half-closed-half-open intervals $[a_n, b_n)$ (always assuming that they are nested)? Justify your answer.