Exercises taken from *Introduction to Probability*, second revised edition, Grinstead and Snell, to be worked out during lectures.

1. (3.1.2) An automobile manufacturer has four colors available for automobile exteriors and three for interiors. How many different color combinations can he produce?
2. (3.1.3) In a digital computer, a bit is one of the integers $\{0, 1\}$, and a word is any string of 32 bits. How many different words are possible?
3. (3.1.5) There are three different routes connecting city A to city B. How many ways can a round trip be made from A to B and back? How many ways if it is desired to take a different route on the way back?
4. (3.1.6) In arranging people around a circular table, we take into account their seats relative to each other, not the actual position of any one person. Show that n people can be arranged around a circular table in $(n-1)!$ ways.
5. (3.1.7) Five people get on an elevator that stops at five floors. Assuming that each has an equal probability of going to any one floor, find the probability that they all get off at different floors.
6. (3.1.8) A finite set Ω has n elements. Show that if we count the empty set and Ω as subsets, there are 2^n subsets of Ω.
7. (3.1.10) A deck of ordinary cards is shuffled and 13 cards are dealt. What is the probability that the last card dealt is an ace?
8. (3.1.13) A certain state has license plates showing three numbers and three letters. How many different license plates are possible
   (a) if the numbers must come before the letters?
   (b) if there is no restriction on where the letters and numbers appear?
9. (3.2.1) Compute the following: $\binom{6}{3}, \binom{26}{26}, \binom{10}{9}$.
10. (3.2.2) In how many ways can we choose five people from a group of ten to form a committee?
11. (3.2.3) How many seven-element subsets are there in a set of nine elements?
12. (3.2.9) Find integers n and r such that the following equation is true:

$$\binom{13}{5} + 2\binom{13}{6} + \binom{13}{7} = \binom{n}{r}.$$

13. (3.2.12) A poker hand is a set of 5 cards randomly chosen from a deck of 52 cards. Find the probability of a
   (a) royal flush (ten, jack, queen, king, ace in a single suit).
   (b) straight flush (five in a sequence in a single suit, but not a royal flush).
   (c) four of a kind (four cards of the same face value).
   (d) full house (one pair and one triple, each of the same face value).
   (e) flush (five cards in a single suit but not a straight or royal flush).
   (f) straight (five cards in a sequence, not all the same suit). (Note that in straights, an ace counts high or low.)

14. (3.2.16) The Siwash University football team plays eight games in a season, winning three, losing three, and ending two in a tie. Show that the number of ways that this can happen is
$$\binom{8}{3}\binom{5}{3} = \frac{8!}{3!3!2!}.$$

15. (3.2.19) A gin hand consists of 10 cards from a deck of 52 cards. Find the probability that a gin hand has
    (a) all 10 cards of the same suit.
    (b) exactly 4 cards in one suit and 3 in two other suits.
    (c) a 4, 3, 2, 1, distribution of suits.

16. (3.2.20) A six-card hand is dealt from an ordinary deck of cards. Find the probability that:
    (a) All six cards are hearts.
    (b) There are three aces, two kings, and one queen.
    (c) There are three cards of one suit and three of another suit.

17. (3.2.22) How many ways can six indistinguishable letters be put in three mail boxes? Hint: One representation of this is given by a sequence $|LL|L|LLL|$ where the $|$'s represent the partitions for the boxes and the L's the letters. Any possible way can be so described. Note that we need two bars at the ends and the remaining two bars and the six L's can be put in any order.

    Remark: it is possible that one or two mail boxes have no letter. For example $|LLL||LLL|$.

18. (3.2.1) Compute $b(5, 0.2, 4), b(8, 0.3, 5)$.

19. (3.2.6) Charles claims that he can distinguish between beer and ale 75 percent of the time. Ruth bets that he cannot and, in fact, just guesses. To settle this, a bet is made: Charles is to be given ten small glasses, each having been filled with beer or ale, chosen by tossing a fair coin. He wins the bet if he gets seven or more correct. Find the probability that Charles wins if he has the ability that he claims. Find the probability that Ruth wins if Charles is guessing.

20. (3.2.15) A baseball player, Smith, has a batting average of .300 and in a typical game comes to bat three times. Assume that Smiths hits in a game can be considered to be a Bernoulli trials process with probability 0.3 for success. Find the probability that Smith gets 0, 1, 2, and 3 hits.

21. (3.2.31) Each of the four engines on an airplane functions correctly on a given flight with probability .99, and the engines function independently of each other. Assume that the plane can make a safe landing if at least two of its engines are functioning correctly. What is the probability that the engines will allow for a safe landing?