SYLLABUS FOR INTRODUCTION TO PROBABILITY (MATH4008)

This is a detailed syllabus for *Introduction to Probability* (MATH4008) at University of Cincinnati, Fall semester, 2014. Below listed are the required reading materials (subtitles) from each chapter of the book. Most parts of the textbook involving simulation are not required for this class. A few sophisticated examples are also not included.

For the listed exercise problems, the solutions for odd-numbered ones are available online. Some of them are also discussed in classroom.

(More comments on the materials may be added at a later moment.)

1. Discrete Probability Distributions

1.1. **Simulation of Discrete Probabilities.** Not required.

1.2. **Discrete Probability Distributions.**

- Random variables and sample spaces.
- Distribution functions.
- Properties.
- Tree diagrams.
- Uniform distribution.
- Infinite sample spaces.
- Historical remarks.
- Exercises: 6, 7, 8, 9, 10, 14, 15, 16, 17 (a), (b), 18 (a), 20, 23, 25*, 27, 28*, 31*.

2. Continuous Probability Distributions

2.1. **Simulations of Continuous Probabilities.** Not required.

2.2. **Continuous Density Functions.**

- Spinners.
- Darts.
- Sample space coordinates.
- Density functions of continuous random variables.
- Cumulative distribution functions of continuous random variables.
- Exercises: 3, 4, 5, 8, 12*, 13.

3. Combinatorics

3.1. **Permutations.**

- Counting problems.
- A counting technique.
- Tree diagrams.
- Birthday problem*.
- Permutations.
- Factorials.
- Exercises: 2, 3, 5, 6, 7, 8, 10, 12, 13, 14.

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3.2. Combinations.

- Binomial coefficients.
- Pascal's triangle.
- Poker hands.
- Bernoulli trials.
- Binomial probabilities.
- Binomial distributions.
- Inclusion-exclusion principle.
- Choosing a sample space.
- Historical remarks*.
- Exercises: 1, 3, 9, 10, 15, 19, 20, 24, 31, 34 (a), 36.

4. Conditional Probability

4.1. Discrete Conditional Probability.

- Conditional probability.
- Bayes probabilities.
- Independent events.
- Joint distribution functions and independence of random variables.
- Independent trials processes.
- Bayes' formula.
- Historical remarks*.
- Exercises: 3, 4, 6, 7, 15, 18, 19, 22, 32, 34, 39, 43.

4.2. Continuous Conditional Probability.

- Independent events.
- Joint density and cumulative distribution functions.
  For this class, students are required to know how to use the joint density and c.d.f. for two random variables ($n = 2$).
- Independent random variables.
- Independent trials.
- Beta density*.
- Exercises: 1, 2, 3, 4, 5, 6.

5. Important Distributions and Densities

5.1. Important Distributions.

- Discrete uniform distributions.
- Binomial distribution.
- Geometric distribution.
- Poisson distribution.

5.2. Important Densities.

- Continuous uniform density.
- Exponential and Gamma densities.
  We do not require Gamma distribution for this class, although this is a very important distribution.
- Functions of a random variables.
- Normal densities.

6. Expected Value and Variance

6.1. Expected Value of Discrete Random Variables.

- Average value.
- Expected value.
- Interpretation of expected value.
- Expectation of a function of a random variable.
- The sum of two random variables.
- Independence.
- Roulette.
- Conditional expectation*.
- Martingales*.
- Historical remarks*.
- Exercises: 1, 2, 3, 4, 5, 14, 17*, 19, 22, 23.

6.2. Variance of Discrete Random Variables.

- Variance.
- Standard deviation.
- Calculation of variance.
- Properties of variance.
- Bernoulli trials.
- Poisson distribution*.
- Exercises: 1, 2, 3, 4, 5, 7, 8, 12, 13, 14, 15.

6.3. Continuous Random Variables.

- Expected value.
- Expectation of a function of a random variable.
- Expectation of the product of two random variables.
- Variance.
- Independent trials.
- Exercises: 1, 2, 8, 10, 17, 19.

7. Sums of Independent Random Variables

7.1. Sums of Discrete Random Variables.

- Convolutions.
  Example 7.2 not required.
- Exercises: 1, 2, 3, 5.

7.2. Sums of Continuous Random Variables.

- Convolutions.
- Sum of two independent uniform random variables.
- Sum of two independent exponential random variables
- Sum of two independent normal random variables.
- Exercises: 1, 2, 3, 4, 5, 6, 7*, 10, 11, 13, 14.

8. Law of Large Numbers

8.1. Law of Large Numbers for Discrete Random Variables.

- Chebyshev inequality.
- Law of large numbers.
- Law of averages.
- Coin tossing.
- Die rolling.
- Numerical comparison.

- Historical remarks.
- Exercises: 1, 5, 6, 7, 8, 10.

8.2. **Law of Large Numbers for Continuous Random Variables.**

- Chebyshev inequality.
- Law of large numbers.
- Uniform case.
- Normal case.
- Exercises: 1, 2, 3, 4, 5, 6, 7.

9. Central Limit Theorem

Focus on Definition 9.1 and Theorems 9.2, 9.4 and 9.6. Theorems 9.1 and 9.3 (so-called local CLTs) are not required.

9.1. **Central Limit Theorem for Bernoulli Trials.**

- Bernoulli trials.
- Standardized sums.
- Approximating binomial distributions.
- Approximation of binomial probabilities.
- Applications to statistics*.
- Historical remarks*.
- Exercises: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12.

9.2. **Central Limit Theorem for Discrete Independent Trials.**

- Standardized sums.
- Approximation theorem.
- Central limit theorem for a discrete independent trials process.
- Examples.
- A more general central limit theorem*.
- Historical remarks*.
- Exercises: 1, 2, 3, 5, 6.

9.3. **Central Limit Theorem for Continuous Independent Trials.**

- Standardized sums.
- Central limit theorem.

10. Generating Functions

10.1. **Generating Functions for Discrete Distributions.**

- Moments.
- Moment generating functions.
- Examples.
- Ordinary generating functions*.
- Properties.
- Heads or tails.
- Exercises: 1, 3 (b) (c), 5, 7, 8.

10.2. **Branching Processes*.**

- Historical background*.
- Problem of extinction*.
- Examples*.

10.3. **Generating Functions for Continuous Densities*.**

- Moments*.
- Moment generating functions*.
- Examples*.
- Exercises: 1, 2, 3, 7, 8, 9, 10.