Name

1. On Halloween, Mr. and Mrs. Smith gave out 100 (indistinguishable) candies to 10 kids from the neighbor area. It might happen that a kid came too late so that the candies had been all gone already.
   (a) How many different ways of distribution could have occurred?
   (b) What about if Mr. and Mrs. had planned well in advance to guarantee that every kid would have at least one candy?
   (c) Describe the two problems in the balls and bars language.

2. Let X be a random variable satisfying $\mathbb{P}(X > x) = x^{-\alpha}$ for $x > 1$ for some $\alpha > 0$. In this case $X > 1$. Find the c.d.f. and p.d.f. of $(2X)^2$.

3. Choose independently two numbers B and C at random from the interval $[0,1]$ with uniform density. Note that the point (B,C) is then chosen at random in the unit square.
   (a) Compute $\mathbb{P}(\min(B,C) > 1/5)$.
   (b) Find the probability density function of $Z = \max(B,C)$.
   (c) Compute $\mathbb{E}Z$ and $\mathrm{Var}Z$.
   (d) Compute $\mathbb{E}(2B+C)$ and $\mathrm{Var}(2B+C)$.

4. Let X be a random variable with probability density function

$$p_X(x) = \begin{cases} cx^2 & x \in [0,1] \\ 0 & \text{otherwise.} \end{cases}$$

   (a) Identify the constant c.
   (b) Compute $\mathbb{E}X$ and $\mathrm{Var}X$.

5. A card is drawn at random from a deck consisting of cards numbered 1 through 9. A player wins 1 dollar if the number on the card is odd and loses 1 dollar if the number if even. What are the expected value and variance of his winnings?

6. Throw a die three times.

   (a) What is the probability that the sum of the first two throws is strictly larger than 8?

   (b) What is the probability that the sum of the three throws is 11?