

Name

The total time for this quiz is 55 minutes. Show steps clearly.

1. We are interested in how to schedule all the regular season games of an NBA team, say San Antonio Spurs, the defending champion. The Spurs have 82 regular season games, with 41 home games and 41 road games. To simplify our consideration, we do not take into account the opponent team that each time the Spurs are playing against. As a fact, Spurs are playing on the road against Lakers this Friday (Nov 14), and before that, they have played 3 home games and 4 road games this season.

How many different schedules the Spurs may have, so that right before playing the 5th road game, they have exactly 3 home games? (Again, no consideration on the opponent teams, nor the game dates.)

2. Let X be a uniform random variable on the interval $(0, 2)$.
 - (a) What are the p.d.f. and c.d.f. of X ?

(b) What are the p.d.f. and c.d.f. of $Y = 1 - X/2$?

3. Suppose we have a die that is not fair; instead, at each throw it shows $1, \dots, 5$ with probability 0.1 each, and 6 with probability 0.5.
- (a) What is the mean and variance of the value of this die (one throw)?
- (b) Throw this die twice and let S denote the sum of the two values. What is the mean and variance of S ?
- (c) Throw this die twice. What is the probability that the sum is 10?

4. Consider two independent random variables B and C , both taking values in $[0, 1]$, with the following p.d.f.s:

$$p_B(x) = 1, p_C(x) = 2x, x \in [0, 1].$$

- (a) Find the p.d.f. of $Y = \max(B, C)$.

- (b) Find the p.d.f. of $Z = B + C$.

Hint: first determine the range of Z . You can use the formula

$$p_Z(z) = \int_{-\infty}^{\infty} p_B(z - x)p_C(x)dx.$$