- 4.2.2

  (a) $\dfrac{\int_1^{10} 0.1e^{-0.1t}dt}{\int_1^{\infty} 0.1e^{-0.1t}dt} = \dfrac{e^{-0.1} - e^{-1}}{e^{-0.1}} = 1 - e^{-0.9}$.

  (b) $\dfrac{\int_5^{10} 0.1e^{-0.1t}dt}{\int_5^{\infty} 0.1e^{-0.1t}dt} = \dfrac{e^{-0.5} - e^{-1}}{e^{-0.5}} = 1 - e^{-0.5}$.

  For the first two parts, one can also get the answer by *memoryless property* of the exponential distribution: $\mathbb{P}(T > s + t \mid T > s) = \mathbb{P}(T > t)$, for all $s, t > 0$ and for T with exponential distribution.

  (c) 1.

  (d) $\dfrac{\int_0^{10} 0.1e^{-0.1t}dt}{\int_0^{20} 0.1e^{-0.1t}dt} = \dfrac{1 - e^{-1}}{1 - e^{-2}}$.

- 4.2.4

  (a) $\dfrac{1}{2}$.

  (b) $\dfrac{\frac{1}{2}\pi \cdot 5^2}{\frac{1}{2}\pi \cdot 10^2} = \dfrac{1}{4}$.

  (c) $1 - \dfrac{1}{4} = \dfrac{3}{4}$. Here $1/4$ is the result from part (b).

  (d) $\dfrac{\pi \cdot 5^2}{\frac{1}{2}\pi \cdot 10^2} = \dfrac{1}{2}$.

- 4.2.6

  We didn't cover conditional probability density function in class. See textbook page 162–163. The midterm exam will not have problems on this concept, although this is a very useful one.

  (a) $p(x|E) = \dfrac{p(x)}{\int_E p(y)dy} = \dfrac{1}{\int_{1/4}^1 dy} = \dfrac{4}{3}$, $x \in (1/4, 1)$ and 0 elsewhere.

  (b) $p(x|E) = \dfrac{p(x)}{\int_E p(y)dy} = \dfrac{e^{-x}}{e^{-1} - e^{-10}}$ for $x \in (1, 10)$ and 0 elsewhere.

  (c) The p.d.f. of the location of the dart is $p(x, y) = \frac{1}{100\pi}, (x, y) \in \Omega$ (see e.g. Example 2.11 from textbook). Note that here we have a random vector instead of a random variable, and the sample space is $\Omega = \{(x, y) : x^2 + y^2 \leq 100\}$. So all the p.d.f.s are from $\mathbb{R}^2$ to $\mathbb{R}_+$. Thus,

  $p(x, y|E) = \dfrac{p(x, y)}{\int_E p(x', y')dx'dy'} = \dfrac{\frac{1}{100\pi}}{\frac{1}{2}} = \dfrac{1}{50\pi}$, for $(x, y) \in \Omega$, and 0 elsewhere.

  (d) $p(x, y|E) = \dfrac{p(x, y)}{\int_E p(x', y')dx'dy'} = \dfrac{1}{1/2} = 2$ for $(x, y) \in (0, 1)$ and $x > y$, and 0 elsewhere.