Families of random sup-measures with long-range dependence

Yizao Wang

University of Cincinnati

AMS Fall Central Sectional Meeting

Self-similarity and Long-range Dependence
in Stochastic Processes

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Joint works with

Olivier Durieu, Stilian Stoev and Gennady Samorodnitsky

Extremes for i.i.d. random variables

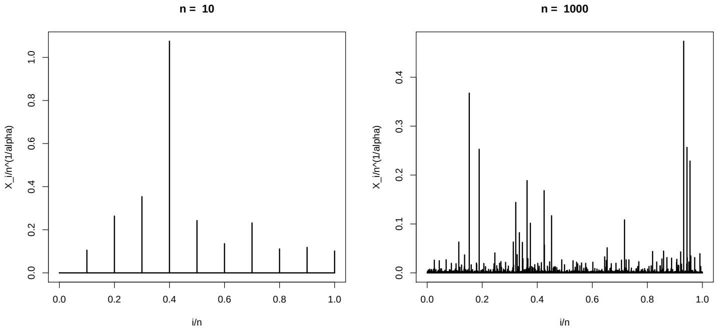
For simplicity, assume

$$\mathbb{P}(X_1 > x) \sim x^{-\alpha}, \text{ as } x \to \infty, \text{ for some } \alpha > 0.$$

Then,

$$\frac{\max_{i=1,\ldots,n} X_i}{n^{1/\alpha}} \Rightarrow \alpha\text{-Fréchet}.$$

Point-process convergence: order statistics and locations



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$$\sum_{i=1}^{n} \delta_{(X_i/n^{1/\alpha},\, i/n)}$$

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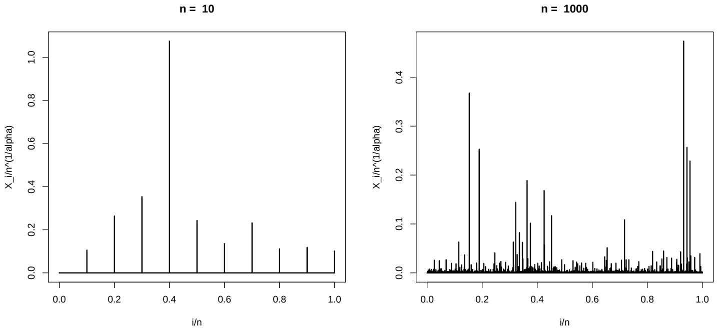

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RHS: a Poisson point process on $\mathbb{R}_+ \times (0,1)$ with intensity measure $\alpha x^{-\alpha-1} dx du$.

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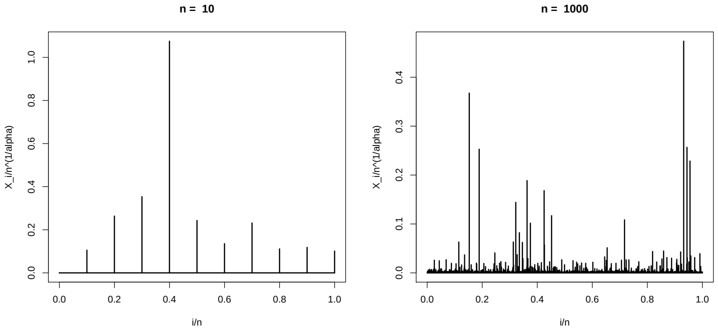

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For an elegant application, see LePage, Woodroofe and Zinn (1981).

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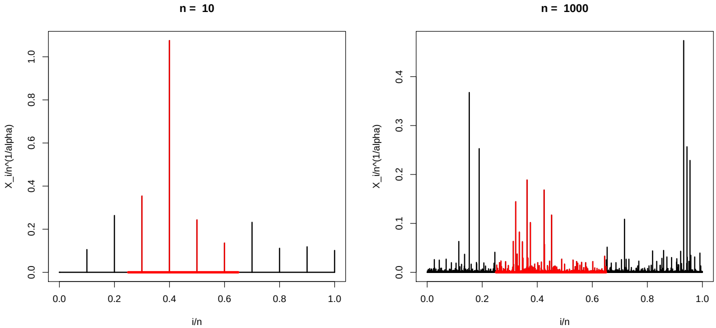
Long-range dependence

- Different order of normalization,
- **O'Brien, Torfs and Vervaat (1990)**: RSM, possibly beyond max-stable,
- very few limit theorems until very recently.

Random sup-measures (RSMs)



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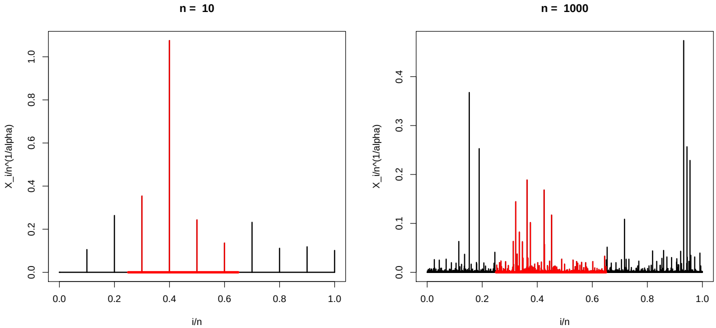

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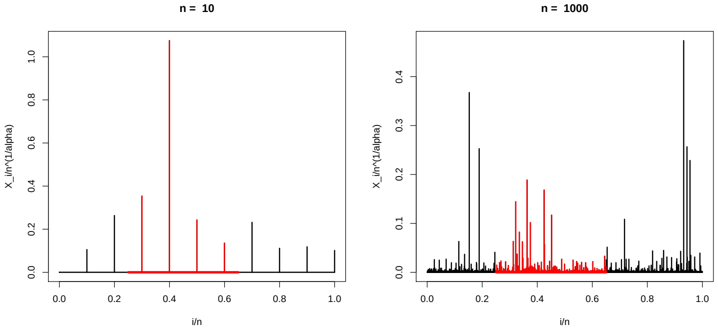

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$$\frac{1}{n^{1/\alpha}} \max_{i/n \in \cdot} X_i \Rightarrow \sup_{i \in \mathbb{N}} \frac{1}{\Gamma_i^{1/\alpha}} 1_{\{U_i \in \cdot\}} =: \mathcal{M}_\alpha(\cdot) \quad \text{independently scattered RSM}$$

in the space of sup-measures. A set function m is a sup-measure, if

$$m(\cup_\lambda A_\lambda) = \sup_\lambda m(A_\lambda).$$

(See Gena's talk.)

Independently scattered RSM

$$\mathcal{M}_\alpha(\cdot) := \sup_{i\in\mathbb{N}} \frac{1}{\Gamma_i^{1/\alpha}} 1_{\{U_i\in\cdot\}}$$

- $\mathcal{M}_\alpha(A)$ is α-Fréchet with scale parameter $\mathrm{Leb}(A)$.
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Will see two examples, where PPP representation characterizes order statistics and corresponding location **sets**.

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Molchanov and Strokorb (2016), Durieu and W (2018)

$$\mathcal{M}^{\mathrm{K}}_{\alpha,\beta}(\cdot) := \sup_{i\in\mathbb{N}} \frac{1}{\Gamma_i^{1/\alpha}} 1_{\left\{\mathcal{R}_i^{(\beta)}\cap\cdot\neq\emptyset\right\}}, \qquad \text{(on } (0,1))$$

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or equivalently (Pitman 2006)

$$\mathcal{R}^{(\beta)} \stackrel{d}{=} \bigcup_{i=1}^{Q_\beta} \{U_i\}, k \in \mathbb{N},$$

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$$\begin{array}{ccccc} \{U\} & \stackrel{\beta\uparrow 1}{\Leftarrow} & \mathcal{R}^{(\beta)} & \stackrel{\beta\downarrow 0}{\Rightarrow} & [0,1] \\ \mathcal{M}_\alpha & \stackrel{\beta\uparrow 1}{\Leftarrow} & \mathcal{M}^{\mathrm{K}}_{\alpha,\beta} & \stackrel{\beta\downarrow 0}{\Rightarrow} & \text{completely dependent, } \alpha\text{-Fréchet.} \end{array}$$

Completely dependent RSM: $\mathcal{M}(A) \equiv Z$ for all $A \neq \emptyset$.

Stable-regenerative RSM

Owada and Samorodnitsky (2016), Lacaux and Samorodnitsky (2016)

$$\mathcal{M}^{\rm sr}_{\alpha,\beta}(\cdot) := \sup_{i\in\mathbb{N}} \frac{1}{\Gamma_i^{1/\alpha}} 1_{\left\{\widetilde R_i^{(\beta)}\cap\cdot\neq\emptyset\right\}},$$

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$\mathcal{M}_\alpha \overset{\beta\uparrow 1}{\Leftarrow} \mathcal{M}^{\mathrm{sr},*}_{\alpha,\beta} \overset{\beta\downarrow 0}{\Rightarrow}$ completely dependent, totally skewed α-stable if $\alpha<1$.

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With $\mathcal{M} \equiv \mathcal{M}_\alpha$, two interesting choices:

- J_* totally skewed α-stable, $\alpha \in (0,1)$,
- J_* Gamma(θ), $\theta > 0$,

$\rightsquigarrow$ Poisson–Dirichlet $(\alpha, 0)$ and $(0, \theta)$ random partitions (Dombry, Ribatet and Stoev 2017, and Shui's talk).

Summary

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Karlin (non-ergodic):

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- Shift-invariant, self-similar.
- Interpolation relations in β.
- Different ergodic properties (Stoev 2008, Kabluchko and Schlather 2010) of stationary max-increment processes

$$\mathcal{M}_t := \mathcal{M}([t, t+1]), t \geq 0.$$

- Sub-max-stable RSM.