A bivariate parametric long-range dependent time series model with general phase

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Outline

Definitions of bivariate LRD series

A bivariate stationary time series is long-range dependent (LRD) if

Time domain: As $n \to \infty$, its autocovariance matrix $\gamma(n)$ satisfies

$$\gamma(n) \sim \begin{pmatrix} R_{11} n^{2d_1-1} & R_{12} n^{d_1+d_2-1} \\ R_{21} n^{d_1+d_2-1} & R_{22} n^{2d_2-1} \end{pmatrix},$$

where $d_j \in (0,1/2)$, $R_{jk} \in \mathbb{R}$, $j,k = 1,2$ ($R_{12} \neq R_{21}$, in general).

Spectral domain: As $\lambda \to 0^+$, its spectral density matrix $f(\lambda)$ satisfies

$$f(\lambda) \sim \begin{pmatrix} g_{11} \lambda^{-2d_1} & g_{12} e^{-i\phi} \lambda^{-(d_1+d_2)} \\ g_{12} e^{i\phi} \lambda^{-(d_1+d_2)} & g_{22} \lambda^{-2d_2} \end{pmatrix},$$

where $g_{jk} \in \mathbb{R}$ and the phase parameter $\phi \in (-\pi/2, \pi/2]$. **Note:** The spectral domain definition contains 6 parameters.

Remark: $\phi = 0 \Leftrightarrow \gamma(n)$ symmetric at the two tails ($R_{12} = R_{21}$).

Bivariate LRD models

- One-sided VARFIMA$(0, D, 0)$

$$Y_n = \begin{pmatrix} (I-B)^{-d_1}\eta_{1,n} \\ (I-B)^{-d_2}\eta_{2,n} \end{pmatrix} = (I-B)^{-D} Q_+ \epsilon_n,$$

$D = \mathrm{diag}(d_1, d_2)$, $\eta_n \sim \mathrm{WN}(0, \Sigma)$, $\Sigma = Q_+ Q_+'$, $BX_n = X_{n-1}$.

- This model satisfies the spectral domain definition with $\phi = \frac{\pi}{2}(d_1 - d_2)$.

- E.g. it allows for symmetry ($\phi = 0$) only if $d_1 = d_2$.

Can we define a model that allows for general phase?

Bivariate LRD - Motivation from real data

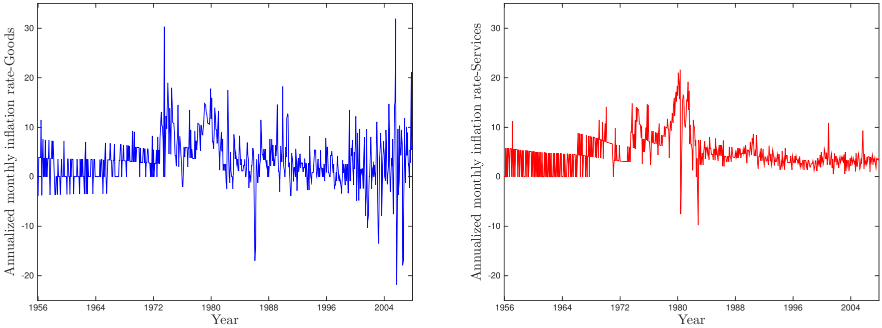


Figure: Annualized monthly U.S. inflation rates for goods (left) and services (right) from February 1956 to January 2008.

Bivariate LRD models

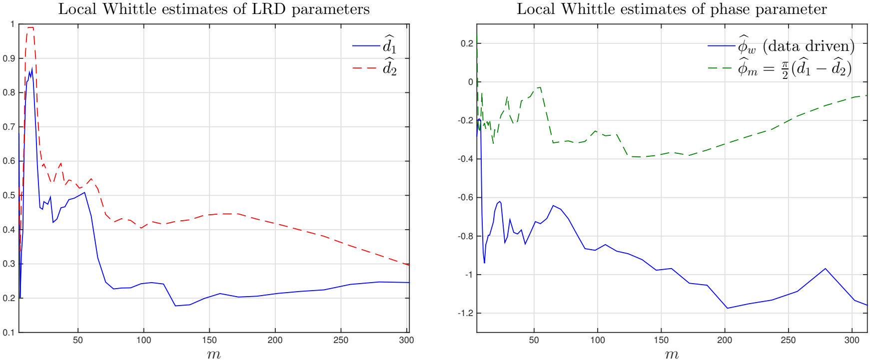


Figure: Left: Local Whittle estimates of d_1, d_2 for the inflation data plotted as functions of a tuning parameter $m = N^{0.25}, \ldots, N^{0.9}$, where N is the sample size. Right: Local Whittle phase estimates, one corresponding to the VARFIMA (dashed line) and one estimated directly from the data (solid line).

Bivariate LRD models

- Two-sided VARFIMA$(0, D, 0)$

$$Y_n = \left((I - B)^{-D} Q_+ + (I - B^{-1})^{-D} Q_-\right) \epsilon_n,$$

where Q_+, Q_- are two real-valued 2×2 matrices.

- This model has a general phase and a closed form $\gamma(n)$.

- The same ϕ can be obtained by more than one choice of Q_+, Q_-.

Can we get a general phase identifiable model?

Bivariate LRD models

- Reduce # of parameters by taking $Q_- = CQ_+, \quad C = \begin{pmatrix} c & 0 \\ 0 & -c \end{pmatrix}$
- Identifiable, general phase, 2-sided VARFIMA$(0, D, 0)$

$$Y_n = \Delta_c(B)^{-1}\eta_n$$
$$\Delta_c(B)^{-1} := (I - B)^{-D} + (I - B^{-1})^{-D} C$$

where $\{\eta_n\} \sim \text{WN}(0, \Sigma)$

- Any $\phi \in (-\pi/2, \pi/2)$ can be obtained by a unique $c \in (-1, 1)$.

What about short memory dynamics?

Bivariate LRD models

- Two-sided VARFIMA(p, D, q) model

$$\Phi(B)X_n = Y_n, \quad Y_n = \Delta_c(B)^{-1}\Theta(B)\eta_n,$$

where $\Phi(B), \Theta(B)$ are the typical AR and MA polynomials.

- General ϕ, identifiability and explicit $\gamma(n)$ remain when $p = 0$.

- Identifiability and form of $\gamma(n)$ are potential issues when $p > 1$.

How should we do inference?

Statistical Inference

LRD model: $\Phi(B)X_n = Y_n, \quad Y_n = \Delta_c(B)^{-1}\Theta(B)\eta_n.$

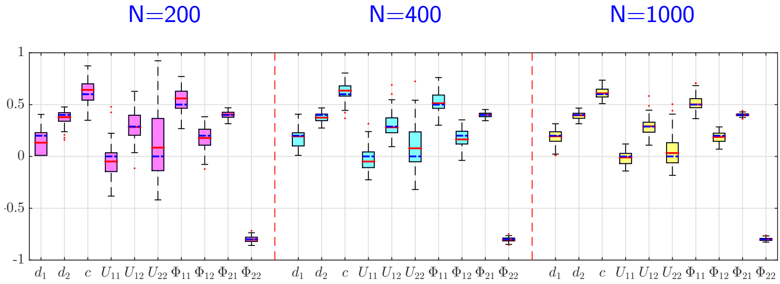
Parameter: $\theta = (d_1, d_2, c, U, \Phi, \Theta)$, $U = \mathrm{chol}(\Sigma)$.

Likelihood: $\widehat{Y}_j = \mathbb{E}(Y_j | Y_1, \ldots, Y_{j-1})$, $V_{j-1} = \mathbb{E}(Y_j - \widehat{Y}_j)(Y_j - \widehat{Y}_j)'$

$$\ell(\theta; Y) \propto -\frac{1}{2}\sum_{j=1}^{N} \log|V_{j-1}| - \frac{1}{2}\sum_{j=1}^{N}(Y_j - \widehat{Y}_j)' V_{j-1}^{-1}(Y_j - \widehat{Y}_j),$$

Conditional MLE: $\widehat{\theta} = \underset{\theta}{\operatorname{argmax}}\ \ell(\theta; Y)$

Simulations



Red lines are medians over 100 replications, blue lines are true parameters

For 2-sided synthetic data with $d_1 = d_2$ and $\phi \neq 0$:

- A misspecified 1-sided model doesn't yield good estimates.
- $\text{AIC}_{\text{2-sided}} < \text{AIC}_{\text{1-sided}}$
- Forecasts are better in terms of MSPE

US inflation data–VARFIMA$(1, D, 0)$

Two-sided

Parameter	$\widehat{d}_1$	$\widehat{d}_2$	$\widehat{c}$	$\widehat{U}_{11}$	$\widehat{U}_{12}$	$\widehat{U}_{22}$	$\widehat{\Phi}_{11}$	$\widehat{\Phi}_{12}$	$\widehat{\Phi}_{21}$	$\widehat{\Phi}_{22}$
Estimate	0.19	0.43	0.33	3.49	0.28	3.38	0.14	0.10	0.05	-0.42
St. error	0.06	0.04	0.17	0.45	0.17	0.51	0.06	0.08	0.02	0.08
p-value	0.00	0.00	0.05	0.00	0.09	0.00	0.02	0.22	0.05	0.00

One-sided

Parameter	$\widehat{d}_1$	$\widehat{d}_2$	$\widehat{c}$	$\widehat{U}_{11}$	$\widehat{U}_{12}$	$\widehat{U}_{22}$	$\widehat{\Phi}_{11}$	$\widehat{\Phi}_{12}$	$\widehat{\Phi}_{21}$	$\widehat{\Phi}_{22}$
Estimate	0.19	0.47		4.60	0.17	2.64	0.13	0.08	0.06	-0.30
St. error	0.05	0.03		0.13	0.11	0.07	0.06	0.08	0.02	0.05
p-value	0.00	0.00		0.00	0.12	0.00	0.04	0.34	0.01	0.00

Conclusions-Future work

What have we done?

- Introduced a new bivariate LRD model
- General phase, identifiability, $\gamma(n)$
- Inference and application to US inflation data

What is next?

- One-sided models, representations
- Multivariate extension

References

- Kechagias, S. and Pipiras, V. (2015), 'Definitions and representations of multivariate long-range dependent time series, *Journal of Time Series Analysis* 36(1), 1-25.
- Kechagias, S. and Pipiras, V. (2016), 'Inference and applications of a bivariate long-range dependent time series model with general phase', *Preprint*.