6th Cincinnati Symposium on Probability
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Titles and abstracts of invited talks

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- *On some possible combinations of mixing rates for strictly stationary, reversible Markov chains*
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- *On the optimality of McLeish's conditions for the central limit theorem*
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- *Busemann processes in positive and zero temperature, on the lattice, and in the continuum*
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- *On the motion of a tagged particle in simple exclusion*
  Sunder Sethuraman, University of Arizona
- *Wandering around the self-normalized limit theory*
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- *Central limit theorems in deterministic dynamical systems*
  Dalibor Volný, Université de Rouen, France
- *Several short stories on quadratic harnesses*
  Jacek Wesołowski, Warsaw University of Technology, Poland
- *Fast algorithms for estimating covariance matrices of stochastic gradient descent solutions*
  Wei Biao Wu, University of Chicago
- *Convergence of the KPZ equation*
  Xuan Wu, University of Illinois, Urbana-Champaign
- *Askey–Wilson signed measures and their applications in open ASEP*
  Zongrui Yang, Columbia University

An invariance principle for de-randomized conductance models

Marek Biskup
University of California, Los Angeles

Abstract. The theory of random walks in disordered reversible environments, a.k.a. conductance models, has advanced remarkably under the assumption that the environment is drawn at random from a law that is stationary and ergodic under translates. Indeed, the concept of the "point of view of the particle" then enables ergodic theorems that conveniently produce limits that would be difficult to establish otherwise and the corrector method from stochastic homogenization yields a proof of diffusive scaling to Brownian motion. Unfortunately, the fact that both the input and the statement are stochastic make it all but impossible to decide whether the stated conclusion applies to a given (non-periodic) conductance configuration. I will show how to overcome this by de-randomizing both the mixing theory for the "point of view of the particle" and the corrector method. An invariance principle will then hold for *every* conductance configuration (modulo certain growth/decay restrictions) whose block averages converge and define an ergodic law on the space of environments.

On some possible combinations of mixing rates for strictly stationary, reversible Markov chains

Rick Bradley
Indiana University, Bloomington

Abstract. The papers of G.O. Roberts and J.S. Rosenthal [*Electronic Comm. Probab.* 2 (1997) 13-25] and G.O. Roberts and R.L. Tweedie [*J. Appl. Probab.* 38A (2001) 37-41] together developed for Markov chains some theory that included (in greater generality) the following "key result": For strictly stationary Markov chains that are reversible and satisfy a certain "irreducibility" condition, "geometric ergodicity" is equivalent to a certain "spectral gap" condition. It is well known that that "key result" can be formulated in terms of the dependence coefficients associated with the absolute regularity (β-mixing) and ρ-mixing conditions. In an exposition of that "key result" that was provided by the author [*J. Time Series Anal.* 42 (2021) 499-533], heavy use was made of a certain rather tight connection, for strictly stationary, reversible Markov chains, between the dependence coefficients associated with the strong mixing (α-mixing) and ρ-mixing conditions. This talk will describe a class of examples constructed in a "follow-up" paper by the author [*Rocky Mountain J. Math.* (accepted for publication)], which shows that for strictly stationary Markov chains that are reversible, the simultaneous mixing rates for the strong mixing and ρ-mixing conditions can (at least within a certain "log convexity" condition) be fairly arbitrary, subject to certain unavoidable tight restrictions. Those examples are countable-state and have the added property that the mixing rate for the absolute regularity condition is within a constant factor of that for strong mixing.

On the optimality of McLeish's conditions for the central limit theorem

Jérôme Dedecker
Université Paris Descartes, France

Abstract. We begin with a historical review of the sufficient conditions for the CLT expressed in terms of conditional expectation, starting with the seminal paper by Gordin (1969). We then show that McLeish's conditions (1975) for the CLT (which can be deduced from Hannan's criterion (1973)) are optimal in a very precise sense. We will also compare our counterexample with other counterexamples in the literature.

Scaling limits for growth driven by reflecting Brownian motion

Amir Dembo
Stanford University

Abstract. In joint works with Kevin Yang, we consider a stochastic Laplacian growth model, that can be viewed as a continuum version of origin-excited random walks. Here, we grow the $(d+1)$-dimensional manifold $M(t)$ according to a reflecting Brownian motion (RBM) on $M(t)$, stopped at level sets of its boundary local time. An averaging principle for the RBM characterizes the scaling limit for the leading order behavior of the interface (namely, the boundary of $M(t)$). This limit is given by a locally well-posed, geometric flow-type PDE, whose blow-up times correspond to changes in the diffeomorphism class of the growing set. Smoothing the interface as we inflate $M(t)$, yields an SPDE for the large-scale fluctuations of an associated height function. This SPDE is a regularized KPZ-type equation, modulated by a Dirichlet-to-Neumann operator. For $d = 1$ we can further remove the regularization, so the fluctuations of $M(t)$ now have a double-scaling limit given by a singular KPZ-type equation.

Some recent advances on limit theorems for stationary random fields

Davide Giraudo
Université de Strasbourg, France

Abstract. In this talk, we will present some recent results obtained by us and various authors on limit theorem for stationary random fields. We will consider partials sums on rectangle of random fields indexed by $\mathbb{Z}^d$ the functional central limit theorem and its quenched version, as well as the law of large numbers and the law of the iterated logarithms. The main tools for these limit theorems are approximation by a multi-indexed martingale or approximation by m-dependent random fields and will be introduced.

Elephant random walks

Allan Gut
Uppsala University, Sweden

Abstract. In the classical simple symmetric random walk the steps are equal to ± 1 and independent. In contrast, in the so called elephant random walk (ERW), introduced by Schütz and Trimper [3] in 2004, "the next step" depends on the whole process so far. This has suggested its name via the fact that elephants have a very long memory.

Our point of departure was Bercu [1], who establishes asymptotics for ERWs using martingale methods. It turns out that the asymptotics depends crucially on the success probability p. More precisely, there is a diffusive regime $(0 \le p < 3/4)$, a superdiffusive regime $(3/4 < p \le 1)$, and a critical one $(p = 3/4)$. Technically, Bercu defines the ERW as a random walk in which the first step, X_1, equals 1 with probability $s \in [0, 1]$ and -1 with probability $1 - s$. After n steps, at position $S_n = \sum_{k=1}^{h} X_k$, the next step is defined as

$$X_{n+1} = \begin{cases} +X_K, & \text{with probability} \quad p \in [0,1], \\ -X_K, & \text{with probability} \quad 1-p, \end{cases}$$

where K has a uniform distribution on the integers $1, 2, \dots, n$.

The aim of our work has been to investigate variations and extensions of ERWs. Main focus has been on cases when the elephant has a restricted memory, such as models in which the elephant remembers only some distant past, only a recent past or a mixture of both. In those settings there is, contrary to the general case, no phase transition. Another idea was to allow for stops, or pauses, at every step with some probability $r = 1 - p - q \in (0, 1)$. In some instances it turns out that the behavior is dramatically different compared to the general case. For more, check our paper [2] and references given there.

Joint work with Ulrich Stadtmüller.

References

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[3] Schütz, G.M., and Trimper, S., Elephants can always remember: Exact long-range memory effects in a non-Markovian random walk. *Phys. Rev. E* **70** (2004), 045101.

Darboux transformation of diffusion processes

Alexey Kuznetsov

York University, Canada

Abstract. Darboux transformation is a well-known technique in the study of second order linear differential operators. Darboux transform of a second order linear differential operator gives another such operator, which is 'almost' isospectral to the first one – the spectrum may be different at most at one point. In other words, this transform allows us to insert or remove eigenvalues in the spectrum of an operator. We will show how to use Darboux transformation to construct new families of diffusion processes whose transition probability densities (and their spectral expansions) are given in closed form. We will also discuss connections to exceptional orthogonal polynomials. This talk is based on joint work with Minjian Yuan.

Global central limit theorems for stationary Markov chains

Michael Lin

Ben-Gurion University, Israel

Abstract. Let $P = P(x, A)$ be a Markov transition probability on a general state space (S, Σ), with invariant probability m. Let $\Omega := S^{\mathbb{N}}$ be the space of trajectories with σ-algebra $\mathcal{A} := \Sigma^{\otimes\mathbb{N}}$, and let $\mathbb{P}_m$ be the probability on $\mathcal{A}$ of the chain with transition probability P and initial distribution m. By invariance of m, $\mathbb{P}_m$ is shift-invariant on $(\Omega, \mathcal{A})$. Let X_n be the projection of Ω on the nth coordinate. Then (X_n) on $(\Omega, \mathcal{A}, \mathbb{P}_m)$ is a stationary Markov chain with state space S. We assume m ergodic for P, so the chain is ergodic too, i.e. the shift on $(\Omega, \mathbb{P}_m)$ is ergodic.

We say that a real centered $f \in L_2(m)$ satisfies the *annealed* CLT if in $(\Omega, \mathbb{P}_m)$ we have

$$\frac{1}{\sqrt{n}} \sum_{k=1}^{n} f(X_k) \xrightarrow{\mathcal{D}} \mathcal{N}(0, \sigma^2), \qquad \text{where} \quad \mathcal{N}(0,0) := \delta_0.$$

We say that a real centered $0 \neq f \in L_2(m)$ satisfies the L_2-*normalized* CLT if

$$\frac{1}{\sigma_n(f)} \sum_{k=1}^{n} f(X_k) \xrightarrow{\mathcal{D}} \mathcal{N}(0, 1),$$

where $\sigma_n(f) := \| \sum_{k=1}^{n} f(X_k) \|_2 > 0$ for large n.

We study conditions which yield that for **every** centered $0 \neq f \in L_2(m)$ a non-degenerate ($\sigma^2 > 0$) annealed CLT and an L_2-normalized CLT hold. Joint work with Christophe Cuny.

On Markov chains based on some new copula families

Martial Longla

University of Mississippi

Abstract. This talk considers Markov chains generated by a new class of bivariate copulas. These copulas depend on some orthogonal functions and their form allows to obtain closed forms of joint distributions and limit theorems for sample averages of functions of the Markov chain. The limiting variance can be computed in closed form, which allows better confidence intervals. Maximum likelihood theory is also developed for some examples. Several examples are constructed to explain the methods and a simulation study is provided.

Around Rosenthal's type inequalities for dependent structures

Florence Merlevède
Université Gustave Eiffel, France

Abstract. Rosenthal inequalities relate moments of order higher than 2 of partial sums of random variables to the variance of partial sums. Besides of being useful to compare the norms $\mathbf{L}^p$ and $\mathbf{L}^2$ of partial sums, these inequalities are important tools for obtaining a variety of results, including tightness of the empirical process, convergence rates with respect to the strong law of large numbers, almost sure invariance principles or $\mathbf{L}^p$-integrated risk for kernel estimators of the density.

Since the 70's, there has been a great amount of works which extended the Rosenthal inequality to dependent sequences. Magda Peligrad has been a leader in this type of extension for a wide range of dependence structures and has led many of her collaborators (including myself) to take an interest in this problem. Among the key results we can cite the following works: Peligrad (1985) and Shao (1995) for the case of ρ-mixing sequences; Shao (1988), Peligrad (1989) and Utev (1991) for the case of ϕ-mixing sequences; Peligrad and Gut (1999) and Utev and Peligrad (2003) for interlaced mixing; Rio (2000) for the strongly mixing case; Rio (2009), Dedecker (2010) and Merlevède and Peligrad (2013) for projective criteria.

In the talk I would like to survey some of these deep results and in particular the Rosenthal inequality that Magda and I obtained in 2013, which reads as follows: Suppose that (X_k) is a real stationary sequence of real-valued random variables that are in $\mathbf{L}^p$, $p > 2$, then

$$\| \max_{1\le j\le n} |S_j| \|_p \le Cn^{1/p}\Big(\|X_1\|_p + \sum_{k=1}^{n} \frac{1}{k^{1+1/p}}\|\mathbb{E}_0(S_k)\|_p + \Big(\sum_{k=1}^{n} \frac{1}{k^{1+2\delta/p}}\|\mathbb{E}_0(S_k^2)\|_{p/2}^{\delta}\Big)^{1/(2\delta)}\Big),$$

where $\delta = \min(1, 1/(p-2))$ and $\mathbb{E}_0(\cdot) = \mathbb{E}(\cdot|\sigma(X_i, i \le 0))$.

This Rosenthal inequality is an important refinement of the Burkholder-like inequality in Peligrad-Utev-Wu (2007). In addition, it is a starting point for many other results including an extension of Rio's inequality (2000) for strong mixing sequences to the larger class of weak α-dependent sequences which leads to new insights in case of intermittent maps of the interval.

Secular coefficients and the holomorphic multiplicative chaos

Joseph Najnudel
University of Bristol, UK

Abstract. We study the coefficients of the characteristic polynomial of unitary matrices drawn from the Circular Beta Ensemble. When the inverse temperature parameter beta is strictly larger than 4, we obtain a new class of limiting distributions that arise when both the order of the coefficient and the dimension of the matrix goes to infinity. For beta equal to 2, we solve an open problem of Diaconis and Gamburd by showing that the middle coefficient tends to zero in probability when the dimension goes to infinity. We introduce a new stochastic object associated to the coefficients of the characteristic polynomial, which we call Holomorphic Multiplicative Chaos (HMC). Viewing the HMC as a random distribution, we prove a sharp result about its regularity in an appropriate Sobolev space. Our proofs expose and exploit several novel connections with other areas, including random permutations, Tauberian theorems and combinatorics.

Revisiting random matrices

Tamer Oraby
University of Texas, Rio Grande Valley

Abstract. In this talk, I will revisit results about random block matrices and products of random matrices and discuss new directions. The work started in my PhD under supervision of Dr. Wlodek Bryc.

Some recent developments on linear processes and linear random fields

Hailin Sang
University of Mississippi

Abstract. In this talk we review some recent developments for the limit theorems and nonparametric estimations of linear random fields with linear processes as their one-dimensional special cases. The linear random fields may have short or long memory. We establish the limit theorems including the local version for linear random fields when the i.i.d. innovations have finite second moment, or the innovations have infinite second moment and belong to the domain of attraction of a stable law with index $0 < \alpha \leq 2$. When the coefficients are absolutely summable we do not have restriction on the regions of summation. However, when the coefficients are not absolutely summable we add the variables on unions of rectangles with some regularity conditions. Entropy is widely applied in the fields of information theory, statistical classification, pattern recognition and so on since it is a measure of uncertainty in a probability distribution. We review some recent work on the estimation of density and quadratic entropy for linear processes with kernel and wavelet methods. We study bias, the integrated square error and central limit theorems for density and entropy estimators with these two methods. We finally propose some open questions on parametric estimations for long memory linear random fields.

Approximating the stationary distribution of the open ASEP

Dominik Schmid
University de Bonn, Germany

Abstract. The exclusion process is one of the best-studied examples of an interacting particle system. In this talk, we consider the stationary distribution of asymmetric simple exclusion processes with open boundaries. We project the stationary distribution onto a subinterval, whose size is allowed to grow with the length of the underlying segment. Depending on the boundary parameters for the exclusion process, we provide sufficient conditions such that the projected stationary distribution is close in total variation distance to a product measure. This talk is based on joint work with Evita Nestoridi.

Busemann processes in positive and zero temperature, on the lattice, and in the continuum

Timo Seppäläinen
University of Wisconsin, Madison

Abstract. This talk is an overview of the construction and properties of multicomponent stationary measures and Busemann functions of random growth models and directed polymer models, and of the utility of these measures in the study of the properties of the models. Potential examples include the corner growth model, the planar directed polymer model, the Kardar-Parisi-Zhang equation, and the directed landscape.

On the motion of a tagged particle in simple exclusion

Sunder Sethuraman
University of Arizona

Abstract. Informally, the simple exclusion process follows a collection of continuous time random walks on $\mathbb{Z}^d$ interacting as follows: When a clock rings, the particle jumps to a nearest-neighbor vertex, if that location is unoccupied. If occupied, the jump is suppressed and clocks start again.

In this model, the motion of a distinguished or 'tagged' particle is of interest. Part of the difficulty in its analysis is that it is not Markovian by itself, due to interaction with other particles.

We will review some of the previous results for the asymptotics of the tagged particle, before coming to recent research on Gumbel limits and large deviations in 1D models, starting from different initial distributions. Based on works with Michael Conroy and SRS Varadhan.

Wandering around the self-normalized limit theory

Qi-Man Shao
Southern University of Science and Technology, China

Abstract. Last two decades has witnessed significant progress on self-normalized limit theory in probability and statistics. In contrast with the classical limit theorems, self-normalized limit theorems, especially Cramér type moderate deviation theorems, require much less moment assumptions. In this talk, we shall review recent developments of limit theory for self-normalized processes as well as applications to statistical inference.

Central limit theorems in deterministic dynamical systems

Dalibor Volný
Université de Rouen, France

Abstract. Any strictly stationary process can be represented in a dynamical system with probability preserving transformation T. If the dynamical system is deterministic (T is of zero entropy) then it seems that none of the known methods of proving the CLT can apply. A CLT can hold true however. We will also show convergence to stable laws. Joint work with Zemer Kosloff.

Several short stories on quadratic harnesses

Jacek Wesołowski
Warsaw University of Technology, Poland

Abstract. In early 1980s several people in Warsaw University of Technology were excited with the question to what extent conditional moments of the first and second order of the Gaussian process uniquely determine the process. Of course, one of the answers gives the celebrated Lévy characterization of the Brownian motion. The idea was to avoid conditions imposed on trajectories though while conditioning with respect to the past-future filtration $(\mathcal{F}_{s,u})_{s<u}$ of the process. It lead Włodek Bryc (and collaborators) to the definition of quadratic harnesses (QH) - the basic object of these short stories: QH is a real-valued stochastic process $(X_t)_{t\geq 0}$ which satisfies two fundamental conditions: for $i = 1, 2$

$$\mathbb{E}(X_t^i \mid \mathcal{F}_{s,u}) \text{ is a polynomial of degree } i \text{ in variables } X_s \text{ and } X_u,\ s < t < u.$$

In the talk I would like to report on several highlights of the mathematical journey of Włodek and his crew (I was lucky to be a member of) in (and around) fascinating land of QHs. These highlights include: infinite integrability; reaching, first, beyond Gaussian and then beyond the Lévy–Meixner processes; non-commutative probability connections; Cauchy kernel families; free Laha–Lukacs theorem; orthogonal martingale polynomials; Askey–Wilson (AW) processes; a near-algebra connection; algebraic structure of polynomial processes;

QH/AW representations of open ASEPs with a view towards KPZ related asymptotics; Motzkin paths asymptotics etc, etc.

And the journey is far from being over...

Fast algorithms for estimating covariance matrices of stochastic gradient descent solutions

Wei Biao Wu
University of Chicago

Abstract. Stochastic gradient descent (SGD), an important optimization method in machine learning, is widely used for parameter estimation especially in online setting where data comes in stream. While this recursive algorithm is popular for the computation and memory efficiency, it suffers from randomness of the solutions. In this talk we shall estimate the asymptotic covariance matrices of the averaged SGD iterates (ASGD) in a fully online fashion. Based on the recursive estimator and classic asymptotic normality results of ASGD, we can conduct online statistical inference of SGD estimators and construct asymptotically valid confidence intervals for model parameters. The algorithm for the recursive estimator is efficient and only uses SGD iterates: upon receiving new observations, we update the confidence intervals at the same time as updating the ASGD solutions without extra computational or memory cost. This approach fits in online setting even if the total number of data is unknown and takes the full advantage of SGD: computation and memory efficiency. This work is joint with Wanrong Zhu and Xi Chen.

Convergence of the KPZ equation

Xuan Wu
University of Illinois, Urbana-Champaign

Abstract. The KPZ universality is a wide class of models for growing interfaces, including the longest increasing subsequence, the Dyson Brownian motion, interacting particle systems and directed polymers. Among these models, the KPZ equation is a canonical member and its asymptotic behaviors have been major topic in this field. In this talk we will give a complete description of the large time limits of the KPZ equation.

Askey–Wilson signed measures and their applications in open ASEP

Zongrui Yang
Columbia University

Abstract. We introduce the Askey–Wilson signed measures as a new tool for studying the stationary measure of the open asymmetric simple exclusion process (ASEP). As applications of this technique, we access several asymptotics of open ASEP: the density profile, limit fluctuations, open KPZ fixed-point limit, half-line ASEP limit, and open ASEP with a light particle. Based on joint works with Dominik Schmid, Yizao Wang and Jacek Wesołowski.

Titles and abstracts of poster presentations

- *On a learning scheme for Value-at-Risk and expected shortfall*
  Juan David Barrera Cano, Universidad de los Andes, Colombia
- *Empirical limit theorems for Wiener chaos*
  Jiemiao Chen, University of Georgia, Athens
- *Cutoff for random walks on contingency tables*
  Zihao Fang and Andrew Heeszel, The Ohio State University
- *Random permutation matrices induced by the Chinese restaurant process*
  Jaime Garza, University of Cincinnati
- *Scaling limit of a stochastic Hegselmann–Krause model*
  Aditya Suresh Gopalan, University of Illinois, Urbana-Champaign
- *Estimation problems for some perturbations of the independence copula*
  Mous-Abou Hamadou, University of Mississippi
- *Non-equilibrium fluctuations for a tagged particle in a one-dimensional zero range processes with regularized random environment*
  Marcel Hudiani, University of Arizona
- *An insight into economic scenario generation for central Africa through time series analysis and copulas*
  Régine Constella Imandi, University of Mississippi
- *Large deviations for the volume of k-nearest neighbor balls*
  Taegyu Kang, Purdue University
- *Asymptotic joint distributions of tree shapes for random phylogenetic trees*
  Gursharn Kaur, University of Virginia
- *Free energy of the elastic random manifold*
  Pax Kivimae, New York University
- *Phase transition of the consistent maximal displacement of branching Brownian motion*
  Jiaqi Liu, University of Pennsylvania
- *On planar Brownian motion singularly tilted through a point potential*
  Barkat Mian, University of Mississippi
- *A point on discrete versus continuous state-space Markov chains*
  Mattias Muia, University of Mississippi
- *Conditional coalescence of a diploid Moran model with selfing*
  Maximillian Newman, Indiana University, Bloomington
- *Weighted average iterated filtering using p-generalized Gaussian smoothing*
  Zamzam Qazi, University of Mississippi
- *Two-steps estimation problem for a new family of copula*
  Sahifa Siddiqua, University of Mississippi
- *Large deviations for empirical measures of self-interacting Markov chains*
  Adam Waterbury, Denison University
- *On the local limit theorems for linear sequences of lower psi-mixing Markov chains*
  Na Zhang, Towson University

On a learning scheme for Value-at-Risk and expected shortfall

Juan David Barrera Cano
Universidad de los Andes, Colombia

Abstract. In this poster we exhibit a regression scheme recently proposed with collaborators to achieve the empirical learning of conditional quantiles and superquantiles from a given random sample. Our results provide convergence guarantees and quantitative upper bounds on the speed of convergence which seem to be realized by the implemented examples and are consistent with the widespread observation that the 'generalization power' of neural networks is preserved under LASSO regularizations.

Empirical limit theorems for Wiener chaos

Jiemiao Chen
University of Georgia, Athens

Abstract. We consider a scheme of central limit theorem for empirical measures. In contrast to a classical empirical central limit theorem where the distribution governing the samples is fixed, we let the distribution change as the sample size grows in a triangular array setup. Certain asymptotic requirement is imposed on the changing distributions in order to create asymptotic uncorrelatedness of the empirical measures evaluated at disjoint subsets. This leads to an independently scattered Gaussian random measure as the limit. We establish weak convergence of multiple integrals with respect to the normalized empirical measures towards multiple Wiener-Itô integrals. This empirical limit theorem is also extended to one involving an infinite series of multiple Wiener-Itô integrals. This is a joint work with Shuyang Bai.

Cutoff for random walks on contingency tables

Zihao Fang and Andrew Heeszel
The Ohio State University

Abstract. Contingency tables are matrices with fixed row and column sums. We studied the Diaconis–Gangolli random walk as a Markov chain Monte Carlo model and observed the cutoff phenomena, a sharp transition from almost deterministic to almost random in the context of mixing time. We gave cutoff time for the random walks on both $1 \times n$ and $n \times n$ contingency tables, and we generalized our method to show cutoff for a family of random walks on the torus $(\mathbb{Z}/q\mathbb{Z})^n$.

Random permutation matrices induced by the Chinese restaurant process

Jaime Garza
University of Cincinnati

Abstract. In this work, we are interested in random permutation matrices whose distribution is induced by the Chinese restaurant process. The Chinese restaurant process is an algorithm depending on two parameters (α, θ) that is used to generate random permutations. For $\alpha = 0$ and $\theta > 0$, permutations generated by the Chinese restaurant process adhere to Ewens distribution with parameter θ. Random permutation matrices sampled using Ewens distribution have been studied extensively in the literature. We complement these results by investigating the linear statistics of the eigenvalues for random permutation matrices induced by the Chinese restaurant process when $\alpha \in (0, 1)$. Joint work with Yizao Wang.

Scaling limit of a stochastic Hegselmann–Krause model

Aditya Suresh Gopalan
University of Illinois, Urbana-Champaign

Abstract. We consider an infinite-dimensional stochastic Hegselmann-Krause model on the real line. In discrete time, each point of a unit-intensity simple point process independently moves halfway toward either of its left or right neighbors, chosen uniformly at random. Co-located points are merged into a single point, and the resulting simple point process is re-scaled to unit intensity. We show that when the point processes are shifted such that there is a point at the origin, there is a unique weak limit of these dynamics when the initial point process is renewal. Using a time-reversal argument, we construct a random positive measure M on $\mathbb{R}$ for a dual problem, and the limiting gap between the first two points is the total mass of M assigned to $\mathbb{R}$. Finally, we discuss some ongoing research directions. Joint work with Partha Dey and Rasoul Etesami.

Estimation problems for some perturbations of the independence copula

Mous-Abou Hamadou
University of Mississippi

Abstract. In this work, we consider symmetric copulas with density of the form:

$$C(u,v) = 1 + \sum_{k=1}^{\infty} \lambda_k \varphi_k(u)\varphi_k(v),$$

where $\{\varphi_k(x), k \in \mathbb{N}\}$ is an orthogonal basis of $L^2(0,1)$ and the sequence $|\lambda_k|$ has a finite number of values or converges to 0. For the finite case, we propose an estimator of the vector $\lambda = (\lambda_1, \ldots, \lambda_s)$ by considering the Markov chains generated by the copula using uniform margins as stationary distribution. In order to determine the confidence interval of parameters, we provide a multivariate central limit theorem for the vector λ.

For $s = 2$, several examples are considered such as the cosine copula, the *sine-cosine copula* and the *Legendre copula*. For each of those copulas, we determine the estimators of the parameters and the central limit theorem as well. A simulation study is provided with a comparison to other known estimators such as MLE and that of Longla and Peligrad (2021).

Non-equilibrium fluctuations for a tagged particle in a one-dimensional zero range processes with regularized random environment

Marcel Hudiani
University of Arizona

Abstract. We prove a non-equilibrium functional central limit theorem for the position of a tagged particle in a one dimensional ε-regularized Sinai random environment zero range process. This is a generalization of the work by Jara-Landim-Sethuraman [Jara et al., 2009] and answers a question in the paper by Landim-Pacheco-Sethuraman-Xue [Landim et al., 2023]. Joint work with Cláudio Landim and Sunder Sethuraman.

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[Landim et al., 2023] Landim, C., Pacheco, C. G., Sethuraman, S., and Xue, J. (2023). On a nonlinear SPDE derived from a hydrodynamic limit in a Sinai-type random environment. *The Annals of Applied Probability*, 33(1):200 – 237.

An insight into economic scenario generation for central Africa through time series analysis and copulas

Régine Constella Imandi
University of Mississippi

Abstract. In this study, we present an Economic Scenario Generator (ESG) tailored for Central African countries, utilizing six variables, including macroeconomic indicators (GDP, GDP deflator inflation, and discount rates), along with three financial variables from BVMAC (The Central Africa Stock Exchange). Our approach incorporates time series analysis, employing ARIMA models to assess variables. To address non-elliptical distributions, we analyze residuals' dependencies using copulas. Subsequently, we use copula results and Cholesky decomposition (specific to elliptical distributions) to project macroeconomic variables. Challenges arise in projecting non-normally distributed financial variables, posing an open problem. The dataset is divided into training and test sets, enhancing the ESG's robustness for economic sector risk management. Joint work with Martial Longla and Louis Aimé Fono.

Large deviations for the volume of k-nearest neighbor balls

Taegyu Kang
Purdue University

Abstract. This paper develops the large deviations theory for the point process associated with the Euclidean volume of k-nearest neighbor balls centered around the points of a homogeneous Poisson or a binomial point process in the unit cube. Two different types of large deviation behaviors of such point processes are investigated. Our first result is the Donsker-Varadhan large deviation principle, under the assumption that the centering terms for the volume of k-nearest neighbor balls grow to infinity more slowly than those needed for Poisson convergence. Additionally, we also study large deviations based on the notion of M_0-topology, which takes place when the centering terms tend to infinity sufficiently fast, compared to those for Poisson convergence. As applications of our main theorems, we discuss large deviations for the number of Poisson or binomial points of degree at most k in a random geometric graph in the dense regime.

This is joint work with Christian Hirsch (Aarhus University) and Takashi Owada (Purdue University).

Asymptotic joint distributions of tree shapes for random phylogenetic trees

Gursharn Kaur
University of Virginia

Abstract. We study two subtree counting statistics, the number of cherries and the number of pitchforks, for random phylogenetic trees generated by two widely used tree models: the proportional to distinguishable arrangements (PDA) and the Yule–Harding–Kingman (YHK) models. We use an extended version of Polya urn models where negative entries are permitted in the replacement matrix. We obtain the strong laws of large numbers and the central limit theorems for the joint distributions of two count statistics: cherries and pitchforks, for the PDA and the YHK models. This is a joint work with Kwok Pui Choi and Taoyang Wu.

Free energy of the elastic random manifold

Pax Kivimae
New York University

Abstract. The elastic random manifold serves as a paradigmatic example of an elastic interface suspended in a quenched disordered medium, with random polymers models being the most well-studied special case. Work in the physics community has suggested that the behavior of these models in high dimensions should enter a number of distinct phases, depending on the temperature, and structure of the disorder, with the famed depinning transition being the most studied.

We confirm a number of predictions for these phases, including existence, rigorously. The central result is a formula for the free energy in the high-dimensional limit, from which we are also able to obtain the behavior of a number of other statistics. The key tool in our computation involves adapting the multi-species synchronization method of Panchenko. Joint work with Gerard Ben Arous.

Phase transition of the consistent maximal displacement of branching Brownian motion

Jiaqi Liu
University of Pennsylvania

Abstract. Branching Brownian motion (BBM) is a random particle system that incorporates both the tree-like structure and the diffusion process. In BBM, each particle moves as Brownian motion and splits into two at rate 1. It is well known that the right-most particle at time t will be near the linear line with slope $\sqrt{2}$. Roberts considered the so-called consistent maximal displacement to answer the question of how close particles can stay to this critical line. We generalize this question and consider how close the trajectory of a particle can stay to the linear line with slope $\sqrt{2}+\varepsilon$ where ε is small and positive. We show that the consistent maximal displacement undergoes a phase transition. This is based on joint work with Julien Berestycki, Bastien Mallein and Jason Schweinsberg.

On planar Brownian motion singularly tilted through a point potential

Barkat Mian
University of Mississippi

Abstract. We will discuss a special family of two-dimensional diffusions, defined over a finite time interval $[0, T]$. These diffusions have transition density functions that are given by the integral kernels of the semigroup corresponding to the two-dimensional Schrödinger operator with a point potential at the origin. Although, in a few ways, our processes of interest are closely related to two- dimensional Brownian motion, they have a singular drift pointing in the direction of the origin that is strong enough to enable the possibly of visiting there with positive probability. Our main focus is on characterizing a local time process at the origin for these diffusions analogous to that for a one-dimensional Brownian motion. Joint work with Jeremy Clark.

A point on discrete versus continuous state-space Markov chains

Mattias Muia
University of Mississippi

Abstract. My poster presents an exploration of the impact of discrete marginals on copula-based Markov chains. We analyze the mixing properties of such models to emphasize the difference between continuous and discrete state-space Markov chains. The Maximum likelihood approach is applied to derive estimators for model parameters in the case of a discrete-state space Markov chain with Bernoulli marginal distribution. A stationary case and a non-stationary case are considered. The asymptotic distributions of parameter estimators are provided. A simulation study showcases the performance of different estimators for the Bernoulli parameter of the marginal distribution. Some statistical tests are provided for model parameters. Joint work with Martial Longla.

Conditional coalescence of a diploid Moran model with selfing

Maximillian Newman
Indiana University, Bloomington

Abstract. We consider a diploid Moran model with selfing with a constant population size. This process generates a random graph consisting of edges connecting parents to children in successive generations, called the pedigree. Classical approaches to studying the random genealogy of a subsample of genes in a population average over the pedigrees. Here we study these random genealogies instead conditioning on the pedigree. We find a robustness of the conditional structure of genealogies to the random dynamics of the pedigree.

Weighted average iterated filtering using p-generalized Gaussian smoothing

Zamzam Qazi
University of Mississippi

Abstract. In recent years, simulation-based inferences have garnered significant attention due to the inherent challenges in directly computing likelihood functions for many real-world problems. Iterated filtering Ionides et al. [2] has emerged as a method to maximize likelihood functions by perturbing models and approximating the gradient of log-likelihood through sequential Monte Carlo filtering. Using Stein's identity, Doucet et al. [1] devised a second-order approximation of the gradient of log-likelihood using sequential Monte Carlo smoothing. In our work, we first introduce the generalization of Stein's identity for normal distribution to p-generalized Gaussian distribution, enabling more flexible perturbation with different tail behaviors. Building upon these gradient approximations, we introduce a novel weighted average algorithm for maximizing likelihood through the two-time-scale stochastic approximation. We show the integration of the algorithm into iterated filtering framework, relaxing the requirement for a bounded variance of the two-timescale stochastic approximation. Initially, we apply p-generalised Gaussian smoothing through Weighted Average Iterated filtering in two toy problems: a linear ou2 model and a nonlinear Gompertz model. Subsequently, we demonstrate the potential of this technique in fitting a more complex cholera model, incorporating a highly nonlinear structure with discrete population dynamics, seasonality, and extra-demographic stochasticity.

References

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Two-steps estimation problem for a new family of copula

Sahifa Siddiqua
University of Mississippi

Abstract. This work delves into continuous state stationary reversible Markov chains generated by a new family of copulas with marginals from a scale-parameter family. A two-step estimation procedure is used to find the estimators of copula parameters. This method requires a consistent estimator of the scale parameter to establish large sample properties of estimator. A simulation study was also done in R to support the findings. Joint work with Martial Longla.

Large deviations for empirical measures of self-interacting Markov chains

Adam Waterbury
Denison University

Abstract. Self-interacting Markov chains arise in a range of models and applications. For example, they can be used to approximate the quasi-stationary distributions of irreducible Markov chains and to model random walks with edge or vertex reinforcement. The term self-interacting Markov chain is something of a misnomer, as such processes interact with their full path history at each time instant, and therefore are non-Markovian. Under conditions on the self-interaction mechanism, we establish a large deviation principle for the empirical measure of self-interacting chains on finite spaces. In this setting, the rate function takes a strikingly different form than the classical Donsker–Varadhan rate function associated with the empirical measure of a Markov chain; the rate function for self-interacting chains is typically non-convex and is given through a dynamical variational formula with an infinite horizon discounted objective function.

On the local limit theorems for linear sequences of lower psi-mixing Markov chains

Na Zhang

Towson University

Abstract. In this project, we investigate the local limit theorem for partial sums of linear sequences of the form $X_j = \sum_{i\in\mathbb{Z}} a_i \xi_{j-i}$. Here $(a_i)_{i\in\mathbb{Z}}$ is a sequence of constants satisfying $\sum_{i\in\mathbb{Z}} a_i^2 < \infty$ and $(\xi_i)_{i\in\mathbb{Z}}$ are functions of a stationary Markov chain with mean zero and finite second moment. The Markov chain is assumed to satisfy one-sided lower psi-mixing condition.