Mixing of Neutral Mesons

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Abstract

This article reviews measurements of mixing in neutral meson systems.

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1 Introduction and Overview

The theory of time-dependent oscillations in the neutral kaon system began with an assertion by Gell-Mann and Pais in 1955 [1]: "It is generally accepted that the microscopic laws of physics are invariant to the operation of charge conjugation (CC); we shall take the rigorous validity of this postulate for granted." At that time, the discovery that weak interactions violate CC symmetry was two years in the future. Nonetheless, the essential insights from their seminal paper hold true: that neutral kaons are produced in strong interactions in two "opposite" flavors, as particle and antiparticle; that the eigenstates of the strong interaction in which flavor is produced and the eigenstates of the weak interaction by which neutral kaons decay differ; that the weak eigenstates are (approximately) equal admixtures of flavor eigenstates; that the lifetimes of the weak neutral eigenstates could differ substantially, and that the “mass difference is surely tiny.” Their prediction that a longer-lived neutral kaon would be observed to decay into three pions was confirmed by Lande, Lederman and Chinowsky [2] in 1957. Today, we denote the strong eigenstates that a longer-lived neutral kaon would be observed to decay into three pions was confirmed by Lande, Lederman and Chinowsky [2] in 1957. Today, we denote the strong eigenstates $K^0$ and $\bar{K}^0$ where the $K^0$ takes its net quantum numbers from its constituent $d$ quark and $\bar{s}$ antiquark. We denote the weak eigenstates $K^0_S$ and $K^0_L$ where the subscripts $S$ and $L$ refer to the eigenstates with the shorter and longer lifetimes. The $K^0_S$ is almost CP-even and the $K^0_L$ is almost CP-odd.

Even before the observation of $K^0_L$ decay to three pions, Pais and Piccioni [3] described how the strong eigenstates of the neutral kaon system oscillate as functions of time, depending on the width and mass differences of the weak eigenstates. Their original summary of the essential physics serves as an excellent example of clarity. Using modern notation for the neutral kaon states, they wrote, “This rather striking prediction about the behavior of the $K^0$ is in some ways similar to the behavior of polarized light under suitable conditions. Circularly polarized light, of either sense of rotation, is a superposition of states of plane polarized light, with planes orthogonal to each other. Conversely, a plane polarized light beam either with horizontal or vertical plane of polarization, is a superposition of two circularly polarized beams with opposite sense of rotation. By selective absorption, a plane-polarized beam can thus be transformed to a circularly polarized beam and vice versa. Quite analogously, the initially produced $K^0$’s transform into $K^0$’s because of a first ‘absorption’ (the decay of the $K^0$’s) and because of a second absorption (the attenuation in nuclear matter of the $\bar{K}^0$’s) transform back into $K^0$’s.”

The formalism originally derived to describe oscillations in the neutral kaon system applies equally to the $D^0 - \bar{D}^0$, $B_d - \bar{B}_d$, and $B_s - \bar{B}_s$ systems. In each case, the time evolution of a neutral system oscillating between its strong interaction eigenstates (also called flavor eigenstates), denoted generically as $P^0$ and $\bar{P}^0$, is described by the Schrödinger equation

$$i \frac{\partial}{\partial t} (P^0(t)) = (M - \frac{i}{2} \Gamma)(\bar{P}^0(t)),$$

where the $M$ and $\Gamma$ matrices are Hermitian, and $CPT$ invariance requires $M_{11} = M_{22} \equiv M$ and $\Gamma_{11} = \Gamma_{22} \equiv \Gamma$. The off-diagonal elements of these matrices describe the dispersive and absorptive contributions to $P^0 - \bar{P}^0$ mixing.

The two eigenstates $P_1$ and $P_2$ of the effective Hamiltonian matrix $(M - \frac{i}{2} \Gamma)$ (also called weak eigenstates) are given by

$$|P_{1,2}\rangle = p|P^0\rangle \pm q|\bar{P}^0\rangle; \quad p^2 + q^2 = 1$$

where $P^0$ and $\bar{P}^0$ are flavor eigenstates with well-defined quark content in the Standard Model. The corresponding eigenvalues are

$$\lambda_{1,2} \equiv m_{1,2} - \frac{i}{2} \Gamma_{1,2} = \left(M - \frac{i}{2} \Gamma\right) \pm \frac{q}{p} \left(M_{12} - \frac{i}{2} \Gamma_{12}\right).$$
where \( m_{1,2}, \Gamma_{1,2} \) are the masses and decay widths and

\[
\frac{q}{p} = \sqrt{\frac{M_{12}^* - \frac{i}{2} \Gamma_{12}}{M_{12} - \frac{i}{2} \Gamma_{12}}} = \left| \frac{q}{p} \right| e^{i\phi} \quad (4)
\]

where \( \phi \) is the relative phase between mixing and direct decay amplitudes when they interfere (see Eqn. (11) below). Unperturbed, the eigenstates of Eqn. (1) evolve as

\[
|P_{1,2}(t)\rangle = e_{1,2}(t)|P_{1,2}\rangle; \quad e_{1,2}(t) = e^{-i(m_{1,2} - \frac{i}{2} \Gamma_{1,2})t}. \quad (5)
\]

As a special and especially important example of the formalism, we consider a state prepared as a flavor eigenstate \(|P^0\rangle\) or \(|\bar{P}^0\rangle\) at \( t = 0 \) where its only interaction is that described by the effective Hamiltonian of Eqn. (1). These states evolve as

\[
|P^0(t)\rangle = \frac{1}{2p} \left[ p(e_1(t) + e_2(t))|P^0\rangle + q(e_1(t) - e_2(t))|\bar{P}^0\rangle \right] \quad (6)
\]

\[
|\bar{P}^0(t)\rangle = \frac{1}{2q} \left[ p(e_1(t) - e_2(t))|P^0\rangle + q(e_1(t) + e_2(t))|\bar{P}^0\rangle \right]. \quad (7)
\]

The decay rate to a final state \(|\alpha\rangle\) at time \( t \) of a particle tagged as a \(|P^0\rangle\) at time \( t = 0 \) is proportional to the product of the magnitude of the transition matrix element squared and the invariant phase space associated with the decay:

\[
d\Gamma \propto |\mathcal{M}|^2 \prod_i \frac{p_i}{E_i} \quad (8)
\]

where the matrix element is defined as \( \mathcal{M} = \langle \alpha | \mathcal{H} | i \rangle \). We will first calculate \( \mathcal{M} \) for \(|i\rangle\) given by Eqn. (6) allowing \(|\alpha\rangle\) to be an arbitrary final state. Later, we will consider specific final states \(|\alpha\rangle\) and we will consider the case where the particle is tagged as a \( P^0 \) at time \( t = 0 \), as described by Eqn. (7).

The equations we derive here differ from those for correlated \( \mathcal{P}\mathcal{P} \) systems such as those produced in \( \Psi(3770) \) decay. In the case of correlated decays the flavors of the two \( P \) mesons are not tagged at \( t = 0 \).

For a neutral particle tagged as a \( P^0 \) at time \( t = 0 \)

\[
\langle \alpha | \mathcal{H} | P^0(t) \rangle = \frac{1}{2} \left[ (A_\alpha + \frac{q}{p} \bar{A}_\alpha)e_1(t) + (A_\alpha - \frac{q}{p} \bar{A}_\alpha)e_2(t) \right] \quad (9)
\]

where we have used the notation

\[
A_\alpha = \langle \alpha | \mathcal{H} | P^0 \rangle; \quad \bar{A}_\alpha = \langle \alpha | \mathcal{H} | \bar{P}^0 \rangle. \quad (10)
\]

The corresponding decay rate is proportional to

\[
|M|^2 \propto \frac{1}{2} e^{-\Gamma t} \left\{ |A_\alpha|^2 \left( \cosh y \Gamma t + \cos x \Gamma t \right) \\
+ |\bar{A}_\alpha|^2 \left( \cosh y \Gamma t - \cos x \Gamma t \right) \\
+ 2 \left[ \Re \left( \frac{q}{p} A_\alpha \bar{A}_\alpha \right) \sinh y \Gamma t - \Im \left( \frac{q}{p} A_\alpha \bar{A}_\alpha \right) \sin x \Gamma t \right] \right\}. \quad (11)
\]

The first term in this expression is the direct decay rate to the final state \(|\alpha\rangle\). It is always the dominant term for sufficiently small decay times. The second term is the mixing rate. Initially, the \( \cosh y \Gamma t \) and \( \cos x \Gamma t \) terms cancel, but over time this term can become dominant. The third term is the
interference term. It depends explicitly on the real and imaginary parts of $A_\alpha$ and $\bar{A}_\alpha$, and on the real and imaginary parts of $q/p$. As for the mixing rate, the interference rate is initially zero, but it can become important at later decay times. Both the mixing and interference rates can be negative, although the overall decay rate is positive definite. The variation of the total decay rate from purely exponential depends on the relative strengths of the direct and mixing amplitudes, their relative phases, the mixing parameters $x$ and $y$, and on the magnitude and phase of $q/p$. In the limit of no direct CP violation ($\bar{A}_\tau = A_\tau$), the phase of $q/p$ becomes the superweak phase [4]:

$$\tan \phi = \tan \phi_{SW} = \left(1 - \left| \frac{q}{p} \right| \right) \frac{x}{y}.$$  

(12)

In the amplitudes denoted $A_\alpha$, $\bar{A}_\alpha$, the final state $|\alpha\rangle$ refers not only to the decay products but also to the point in phase space where they are produced. As a result, the mixing rate can be a strong function of this position. For example, when we consider $D^0 \to K^+\pi^-\pi^0$ the direct decay amplitude will be doubly Cabibbo suppressed, the mixing amplitude will be Cabibbo-favored, and the interference term will produce both enhanced and suppressed rates depending on Dalitz plot position.

As a second example of mixing formalism we consider pairs of neutral mesons produced in $e^+e^-$ collisons at the $\phi(1020)$, the $\Psi(3770)$, the $\Upsilon(4S)$, and the $\Upsilon(5S)$. We write the correlated amplitude for the $P^0$ and $\bar{P}^0$ to decay to the states $\alpha$ and $\beta$ at times $t_1$ and $t_2$ respectively, where the times are measured in the center-of-mass (CM) system and $t = 0$ is the time of the $e^+e^- \to P^0\bar{P}^0$ production. Because the final states are $J^{PC} = 1^{--}$, we need to antisymmeterize the amplitude with respect to charge conjugation. The matrix element is

$$\mathcal{M} = \frac{1}{\sqrt{2}} \left[ \langle \alpha | H | P^0(t_1) \rangle \langle \beta | H | \bar{P}^0(t_2) \rangle - \langle \beta | H | P^0(t_2) \rangle \langle \alpha | H | \bar{P}^0(t_1) \rangle \right]$$  

(13)

which can be re-written as

$$2\sqrt{2} \mathcal{M} = \left( \frac{q}{p} \bar{A}_\alpha A_\beta - \frac{p}{q} A_\alpha \bar{A}_\beta \right) \left[ e_1(t_1)e_2(t_2) - e_1(t_2)e_2(t_1) \right]$$  

(14)

whis has the form

$$2\sqrt{2} \mathcal{M} = X(e_{11}e_{22} - e_{12}e_{21}) + Y(e_{11}e_{22} + e_{12}e_{21}).$$  

(15)

Using $\Delta t = t_1 - t_2$, the correlated decay rate is proportional to

$$|\mathcal{M}|^2 = \frac{1}{8} e^{-\Gamma(t_1+t_2)} \times \{ YY^* (\cosh y \Gamma \Delta t + \cos x \Gamma \Delta t)$$

$$+ XX^* (\cosh y \Gamma \Delta t - \cos x \Gamma \Delta t)$$

$$- 2 [ \Re(XX^*) \sinh y \Gamma \Delta t - \Im(YY^*) \sin x \Gamma \Delta t ] \}$$  

(16)

The terms in this equation parallel those of Eqn. (11): the first is the direct decay rate where the particle decays to $\alpha$ and the antiparticle to $\beta$, or vice versa; the second is the mixing rate; the third is the interference rate. The direct decay amplitude

$$Y = A_\alpha \bar{A}_\beta - \bar{A}_\alpha A_\beta$$  

(17)

has exactly the form one expects naively. The mixing amplitude

$$X = \frac{q}{p} A_\alpha \bar{A}_\beta - \frac{p}{q} A_\alpha A_\beta$$  

(18)
the 30-inch Berkeley propane bubble chamber to detect is also easy to interpret: either the initial particle has “mixed” into antiparticle (giving the term) or vice versa. The coefficients $q/p$ and $p/q$ reflect the possibility that the weak eigenstates are not equal admixtures of the flavor eigenstates. Note that the variation of the decay rate from exponential depends on the decay time difference $\Delta t$ while the argument of the exponential is $t_1 + t_2$. Because the decay rate is odd in $\Delta t$, the interference terms cancel in time-integrated rates.

Table 1 summarizes our current knowledge of the mixing parameters for the various neutral meson systems. The values of the raw and normalized mass and width differences vary by orders of magnitude. As a result, the phenomenology and the experimental approaches differ radically. The interplay of mixing and $CP$ violation is a critical element of many measurements, but the focus of this review is explicitly the mixing parameters themselves. Reviews of $CP$ violation can be found in Refs. [xxxx]. Sections ??, ??, ??, and ?? describe mixing physics and parameter measurements in the neutral kaon, charm, $B_d$ and $B_s$ systems respectively, followed by a summary in Section ??.

Details of the formalism and notation are collected in Appendix ??.

## 2 Mixing in the $K^0$-$\bar{K}^0$ System

In their paper describing how neutral kaon systems oscillate in time, Pais and Piccioni [3] also described an experimental configuration to explicitly demonstrate the quantum mechanical nature of the neutral kaon system. Their technique exploits the fact that the strong interaction cross-sections of the neutral kaon mass difference use $K^0$ beams regenerated by coherent transmission, so we discuss the $K^0$-$\bar{K}^0$ system first.

The first published measurement of the neutral kaon mass difference, by R.H. Good et al. [5], used the 30-inch Berkeley propane bubble chamber to detect $K^0_S$ decays following regeneration from a $K^0_L$ beam incident on a thick metal plate. From the angular distribution of the decays with respect to the incident beam direction, they observed regeneration by (inelastic) scattering from single nucleons, by diffractive (elastic) scattering from complex nuclei, and by coherent transmission through the plate. Diffractive scattering from a nucleus leaves the nucleus in the same state after the collision as it was before. The $K^0$ and $\bar{K}^0$ components of the $K^0_L$ are scattered in the forward direction and the relevant cross-sections are those for $K^+$ and $K^-$ scattering at the same energy, in accordance with charge independence for nuclear cross-sections. Regeneration via transmission over macroscopic lengths is sensitive to the difference between $K^0_S$ and $K^0_L$ masses. Many precise measurements of the neutral kaon mass difference use $K^0_S$ beams regenerated by coherent transmission, so we discuss the key concepts here.

An incident $K^0_L$ beam interacts with target nuclei as a superposition of $K^0$ and $\bar{K}^0$. These two components scatter with different cross-sections, producing some $K^0_S$ as a result. The forward scattering $K^0_S$ amplitudes produced in diffractive scattering from individual nuclei interfere coherently along the entire flight path in the regenerator. Because forward diffractive scattering is elastic, the energies

<table>
<thead>
<tr>
<th>system</th>
<th>$\Delta M$ (MeV)</th>
<th>$\Delta \Gamma$ (MeV)</th>
<th>$\Gamma$ (MeV)</th>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K$</td>
<td>$3.682 \times 10^{-12}$</td>
<td>$7.364 \times 10^{-12}$</td>
<td>$3.483 \times 10^{-12}$</td>
<td>0.946</td>
<td>1.000</td>
</tr>
<tr>
<td>$D$</td>
<td>$10.4 \times 10^{-12}$</td>
<td>$23.8 \times 10^{-12}$</td>
<td>$1.61 \times 10^{-9}$</td>
<td>$0.65 \times 10^{-2}$</td>
<td>$0.74 \times 10^{-2}$</td>
</tr>
<tr>
<td>$B_d$</td>
<td>$3.34 \times 10^{-10}$</td>
<td>--</td>
<td>$4.33 \times 10^{-10}$</td>
<td>0.77</td>
<td>--</td>
</tr>
<tr>
<td>$B_s$</td>
<td>$1.17 \times 10^{-8}$</td>
<td>$(4.1_{-2.4}^{+2.2}) \times 10^{-11}$</td>
<td>$4.62 \times 10^{-10}$</td>
<td>26.2</td>
<td>--</td>
</tr>
</tbody>
</table>
of the incident $K^0_L$ and scattered $K^0_S$ amplitudes are equal. Because the masses differ, the momentum of the produced $K^0_S$ differs from that of the incident $K^0_L$ by a small amount

$$\Delta k = k_1 - k_2 = \frac{-mc^2}{(\hbar c)^2 k}(m_1 - m_2)c^2$$

where the subscripts 1 and 2 refer to $K^0_S$ and $K^0_L$ and where $m$ and $k$ can be taken as the mass and momentum of either meson. Following the discussion in Ref. [3], one can call $\Lambda = \gamma v \tau_1$ the mean path of the $K^0_S$ and introduce the dimensionless quantities $\ell = L/\Lambda$ and $\delta = (m_2 - m_1)c^2/(\hbar/\tau_1)$ where $L$ is the pathlength in the regenerator. The intensity of the undeflected regenerated $K^0_S$ wave, ignoring several complications, is then

$$|\alpha(\ell)|^2 = \left|\frac{f_{21}^0(N^2\Lambda)^2}{\delta^2 + \frac{1}{2}}e^{-i\delta \ell} - e^{-\ell/2}\right|^2e^{-NL\sigma_T}$$

where $N$ is the number of nuclei per cubic centimeter, $\lambda$ is the wavelength of the incident kaon, $\sigma_T$ is the total cross-section, and $f_{21}^0$ is the difference between $K^0$ and $\overline{K}^0$ forward amplitudes. The same formula had been obtained previously by M.L. Good [6, 7]. Were this formula valid, one could simply measure the ratio of transmission regeneration to regeneration by nuclear diffraction to determine $\delta$. The dependences on $f_{21}^0$ would cancel.

![Figure 1](image_url)

**Figure 1:**

In a real experiment, the length of the regenerator plate is long enough compared to the $K^0_S$ decay length that one must account for decay before the end of the plate. Similarly, the probability that the incident $K^0_L$ interacts more than once is comparable to the probability that it interacts only once. Accounting for these complications, the ratio of transmission regeneration to diffractive nuclear regeneration is no longer independent of $f_{21}^0$. R.H. Good *et al.* [5] calculated these corrections and used their data to find $\delta = 0.84^{+0.29}_{-0.22}$. Figure 1 shows the angular distribution of regenerated $K^0_S$ candidates produced in thick (6 inch) and thin (1 1/2 inch) iron plates satisfying selection criteria described in detail in the paper. The curve shows the prediction for $\delta = 0$ which corresponds to no transmission regeneration. The excess in the bins with $0.997 < \cos \theta < 1.000$ are ascribed to coherent transmission regeneration and provide the evidence for $\delta > 0$.

The second published measurement of the neutral kaon mass difference, by V.L. Fitch *et al.* [8], produced a $K^0$ beam and observed the $\overline{K}^0$ component of the $K^0 - \overline{K}^0$ admixture as a function of decay time. $K^0$ mesons were produced at an angle of 84° by 5.3 GeV protons impinging on a platinum
target. Interference effects are damped with a time constant equal to twice the $K_S^0$ lifetime, see Eqn. ??, corresponding to a distance of 2.4 inches for a kaon with momentum 500 MeV/c, a typical momentum for this experiment. To observe the $K^0$ signal so close to the production point, where backgrounds were high, charge exchange in converter material was exploited to produce $K^-$ mesons. These were transported through a magnetic spectrometer to a region approximately 18 ft from the target where kaons were distinguished from pions using a series of Cerenkov and $dE/dx$ counters. The converter distance was varied to sample the neutral kaon composition at 8 decay times ranging from 1 - 4 $K_S^0$ lifetimes. The variation in the detected $K^-$ to $K^+$ ratio as a function of converter distance, shown in Fig. 2, provided evidence for $K^0$ oscillating into $\overline{K}^0$. Because there were discrete converter distances, the data were consistent with many high values for the mass difference. For $\delta < 10$, the paper reports that the best fits yielded $\delta = 1.9 \pm 0.3$ “where most of the error arises from the uncertainty in the velocity.”

Figure 3: “The time distribution of the $\overline{K}^0$ interactions is compared with the expected distributions” for $\delta = 0.75$ and $\delta = 1.50$. From Ref. [9].
The first two measurements published in journal articles, Refs. [5] and [8], were submitted for publication within a week of each other in June 1961. A third group, Camerini et al. [12], reported in 1962, used the 30-inch Berkeley propane bubble chamber exposed to a 790 MeV/c beam. The ∆K0 mass difference was determined to be \( \delta = 0.5292 \pm 0.009 \) [?]. The result was consistent with that of Fitch et al. [11], observed a single candidate for \( N^0 \rightarrow n\pi^+\nu \) decays but forbids \( K^0 \rightarrow e^\pm\pi^\mp\nu \) decays [11]. The first experimental test of this rule by Ely et al. [12], reported in 1962, used the 30-inch Berkeley propane bubble chamber exposed to a 790 MeV/c beam. Charge exchange interactions produced \( K^0 \) mesons which were observed to decay to \( e^\pm\pi^\mp\nu \) or \( e^\mp\pi^\pm\nu \). The time distributions of these decays were compared, and the results found to be incompatible with the selection rule and mixing for 1 < \( \delta \) < 2. They said that their data was consistent with ∆Q = ∆S “only if values of [δ] larger than 5 are assumed”. Additional investigations of the ∆Q = ∆S rule was reported later that year. Barbero-Galtieri et al. [13] observed a single candidate for the ∆Q = ∆S violating decay \( \Sigma^+ \rightarrow n\mu^+\nu \) in an emulsion exposed to a K+ beam. Alexander et al. [14] used a hydrogen bubble chamber to study neutral kaon semileptonic decays and reported that their observations supported the results of Ely et al., although they were also consistent with ∆Q = ∆S and a statistical fluctuation of chance 6.5%. Two years later, Aubert et al. [15] reported results from a higher statistics experiment using the Ecole Polytechnique Heavy Liquid Bubble Chamber at CERN. They found the ∆Q = ∆S violation parameter to be about 10 times less than that reported by Ely et al. and consistent with zero. Assuming this to be so, they used the lepton sign to tag the flavor of the neutral kaon when it decayed and found the mass difference to be \( \delta = 0.78 \pm 0.20 \), compatible with results reported earlier from regeneration experiments. Today, amplitudes describing the violation of the ∆Q = ∆S rule in neutral kaon decays are constrained experimentally to be less than 1% [?].

In 1963, an experiment performed at the Brookhaven National Laboratory reported anomalous regeneration of \( K^0_S \) mesons [16]. A \( K^0_L \) beam with average momentum 1 GeV/c and \( \sigma \approx 300 \) MeV/c impinged a hydrogen bubble chamber about 2.5 m from the production target. “The chamber was cylindrical, 35 cm in diameter, 20 cm deep, with axis parallel to the floor. ...” [The fiducial region comprised a volume of about 4.4 liters, less than 25% of the total chamber volume. In this volume
252 events were found which fitted \([K^0_L]\) decay kinematics.” The angular distribution of the sample observed with \(\cos \theta > 0.980\) is shown as Fig. 4. “The events in the very sharp peak in the forward direction are interpreted as resulting from coherent production. The probability that the peak arises purely as a statistical fluctuation is \(\approx 10^{-6}\). The experimental resolution is estimated to be 1.5° and the width of the peak is consistent with zero within this error. The solid line represents the contribution to be expected from\([K^0_L]\) semileptonic decays. Using calculations for coherent production in hydrogen based on the work of M.L. Good [6, 7], the authors predicted less than 1 event from this source in the highest \(\cos \theta\) bin. Guided by the work of R.H. Good et al. [5], they predicted less than two events in the forward peak from coherent production in the 1.6 cm thick copper walls of the chamber. They further rejected the possibility that the signal originated in two-pion decays of the \([K^0_L]\). This level of \(CP\)-violation was excluded by the prior observations of 411 \([K^0_L]\) decays in cloud chambers, none of which were consistent with two-pion decays. The authors concluded that the results of their experiment suggested the existence of anomalous coherent production of \([K^0_L]\) mesons from a \([K^0_L]\) beam. But they also acknowledged that the physics implications of such a conclusion were extreme, saying “However, in view of the extraordinary consequences which may be required by such a result, it is necessary to emphasize that we cannot, at this time, completely exclude the possibility or even evaluate precisely the probability that the striking character of the data results from a combination of real effects underestimated by us together with strong statistical fluctuation.”

The following year, 1964, brought with it the observation by Christenson, Cronin, Fitch, and Turlay [17] of the \(CP\)-violating two-pion decay of the \([K^0_L]\). Cronin and Fitch were awarded the Nobel Prize in Physics in 1980 for this discovery. Their experiment was designed to accumulate relatively high statistics and to reduce possible backgrounds. The original paper describes that details very well. Using modern notation, they wrote,

“In this measurement, \([K^0_L]\) mesons were produced at the Brookhaven AGS in an internal Be target bombarded by 30-GeV protons. A neutral beam was defined at 30 degrees to the circulating protons by a \(1\frac{1}{2}\)-in. \(\times\) \(1\frac{1}{2}\)-in. \(\times\) 48-in. collimator at an average distance of 14.5 ft. from the internal target. This collimator was followed by a sweeping magnet of 512 kG-in. at 20 ft. and a 6-in.\(\times\) 6-in.\(\times\) 48-in. collimator at 55 ft. A \(1\frac{1}{2}\)-in. thickness of Pb was placed in front of the first collimator to attenuate the gamma rays in the beam.”
The experimental layout is shown in relation to the beam in Fig. 1 [Fig. 5]. The detector for the decay products consisted of two spectrometers each composed of two spark chambers for track delineation separated by a magnetic field of 178 kG-in. The axis of each spectrometer was in the horizontal plane and each subtended an average solid angle of $0.7 \times 10^{-2}$ steradians. The spark chambers were triggered on a coincidence between water Cherenkov and scintillation counters positioned immediately behind the spectrometers. When coherent $K^0_S$ regeneration in solid materials was being studied, and anticoincidence counter was placed immediately behind the regenerator. To minimize interactions $K^0_L$ decays were observed from a volume of He gas at nearly STP."

The signature for two-body decay of $K^0_L$ mesons was the observation of a peak in the very forward direction of candidate two-body decays with the neutral kaon mass. Fig. 6 shows the angular distribution for candidates in the neutral kaon mass region as well as the distributions for the mass regions just below and above. The characteristics of the events in the peak were compared with those produced from coherent regeneration in tungsten and found to “appear identical”. The net signal was determined to be $45 \pm 9$ events, yielding a branching ratio $R = (K^0_L \rightarrow \pi^+ \pi^-)/(K^0_L \rightarrow \text{all charged modes}) = (2.0 \pm 0.4) \times 10^{-3}$.

Data also were taken with a hydrogen target, producing a similar forward peak with $45 \pm 10$ events. The background from coherent regeneration was estimated to be $\approx 10$ events. The net signal was judged to be entirely consistent with the $CP$-violating two-pion decay rate of the $K^0_L$ measured in the helium volume and more than 15 times that expected extrapolating the anomalous regeneration rate reported by Leipuner et al. [16]. Later experiments have exploited the coherent interference of the $K^0_L \rightarrow \pi^- \pi^+$ amplitude and a regenerated $K^0_S \rightarrow \pi^- \pi^+$ amplitude to make the most precise measurements of $\Delta m$, as discussed below.

Following up their observation of the two-pion decay of $K^0_L$, Christenson et al. reported results of a detailed study of $K^0_S$ regeneration from $K^0_L$ beams and the $K^0_L - K^0_S$ mass difference [18]. The apparatus used for these measurements was the same as that described in Ref. [17] with the addition of regenerators made from a variety of materials including C, Fe, W, H, and full- and half-density Cu placed 58 ft. (300 mean $K^0_S$ decay lengths) from the neutral kaon production target. The mean momentum of the kaon beam was 1.1 GeV/c with "a spread of $\pm 100$ MeV/c."

In measuring $\Delta m$, they introduced the use of two regenerators to eliminate systematic uncertainties due to poor knowledge of relevant nuclear scattering parameters. As a very good first approximation, the total regeneration amplitude following the second regenerator is the sum of the two regeneration amplitudes with the first diminished to account for decay in flight and with relative phase $\delta g$ where $g \equiv G/\Lambda_1$ is the physical gap distance $G$ normalized to the $K^0_S$ decay length $\Lambda_1$:

$$A(G) = a_1 e^{\phi_1} e^{ig} e^{-ig/2} + a_2 e^{i\phi_2}.$$  (21)

The integrated $K^0_S$ intensity following the second regenerator therefore depends on the gap distance
and the mass difference:

\[ I(G) = |A(G)|^2 = a_1^2 e^{-g} + a_2^2 + 2a_1a_2 e^{-g/2} \cos(\delta g + \phi_0) \, . \]  

The last term represents the interference of the two regeneration amplitudes. While the magnitude of this otherwise exponential decay term depends on the nuclear cross sections, its oscillatory behavior does not.

When the experiment was designed, the authors anticipated \( \delta \) in the range 1.5 to 2.0, so optimized the lengths of the regenerators and the gaps accordingly. Data were taken for seven gap distances, and the mass difference was measured to be \( \delta = 0.50 \pm 0.10 \). This value is consistent with the currently accepted value and with that originally reported by R.H. Good et al. [5] but not with the values concurrently reported by Fitch et al. [8] or subsequently by Camerini et al. [9].

In addition to measuring the mass difference, the authors studied the regeneration mechanism in detail. They created a "half-dense" target with twelve \( \frac{1}{8} \)-inch slices of copper separated by \( \frac{1}{5} \)-inch air gaps and compared regeneration in this target with that in a solid 3-inch block of copper. They used the angular distributions of the regenerated \( K^0 \) mesons to extract the incoherent and coherent regeneration signals. The also studied the \( A \)-dependence of coherent regeneration by varying regenerator material, including hydrogen. This work validated the theory of regeneration in detail, and established its use as a powerful tool for precision kaon measurements for the next 40 years.

Because the first two experiments using the explicit time-dependence of flavor oscillations [8, 9] measured values of \( \Delta m \) much greater than that measured in the regeneration experiments, it was important that the results from later experiments using both techniques converged on a common value. In 1966 Camerini et al. [20] reported a measurement of \( \Delta m \) using the time-dependence of \( \overline{K}^0 \) mesons oscillating into \( K^0 \) mesons. The \( \overline{K}^0 \) mesons were produced via charge exchange scattering of \( K^- \) incident on hydrogen and deuterium in a 25-inch bubble chamber. The \( K^0 \) mesons were
observed when they interacted downstream producing either a Λ or Σ baryon. The mass difference was determined to be $\delta = 0.50 \pm 0.15$ from the proper time distribution of the signal events. The authors collated and averaged the results of previous experiments. Table 2 is adapted from Table I in Ref. [20]. It includes the results of Refs. [5, 8, 9, 15], and [18] discussed above along with an erratum (never published) for Ref. [9] and results from several other experiments [21, 19, 22, 23]. Discarding the results of Fitch et al. [8], they combined the measurements of $\Delta m$ extracted from the time dependence of strangeness oscillations tagged using hadronic interactions to give $\delta = 0.63 \pm 0.11$ and from regeneration experiments to give $\delta = 0.51 \pm 0.04$. They concluded that, “there is no reason to believe that the value of the mass difference depends on the method employed.”

Accounting for the $CP$ violating $K_L^0 \to \pi^- \pi^+$ amplitude became important in extracting $\Delta m$ with greater precision. And measuring $\Delta m$ with greater precision was necessary to extract the parameters describing $CP$ violation precisely. Given a pure $K_L^0$ beam, the amplitudes for regenerated $K_S^0 \to \pi^- \pi^+$ and for $K_L^0 \to \pi^- \pi^+$ decays interfere coherently to produce the $\pi^- \pi^+$ signal following a regenerator plate. The ratios of $K_L^0$ and $K_S^0$ amplitudes to the $\pi \pi$ final states are denoted

$$
\eta_{+-} = \frac{\langle \pi^- \pi^+ |K_L^0\rangle}{\langle \pi^- \pi^+ |K_S^0\rangle} \quad \text{and} \quad \eta_{00} = \frac{\langle \pi^0 \pi^0 |K_L^0\rangle}{\langle \pi^0 \pi^0 |K_S^0\rangle}.
$$

(23)

The magnitudes of $|\eta_{+-}|$ and $|\eta_{00}|$ are $\approx 2 \times 10^{-3}$. The most direct way to measure the phase of $\eta_{+-}$ ($\eta_{00}$) is to observe the interference of the $K_S^0$ and $K_L^0$ amplitudes. Many experiments were done in the next few years to measure $\eta_{+-}$ and $\Delta m$, and it is not possible to discuss all of them. We discuss several measurements of $\Delta m$ which either introduced new approaches or achieved notable levels of precision.

Cullen et al. [24] exposed the regenerator configuration illustrated in Fig. 7 to a neutral beam produced from the external target of the CERN Proton Synchrotron. They detected $\pi^- \pi^+$ decays in the regions labelled $N_1$, $N_2$, and $N_{12}$ using a large acceptance wire chamber spectrometer. Background levels were determined with 0.5% precision. By design, the zero-crossing point for the difference of
rates \( \frac{N_{12} - N_1 - N_2}{2\sqrt{N_1N_2}} \) was sensitive to \( \Delta m \) but not to \( \eta_{+-} \) or to the \( K_S^0 \) lifetime. They ran with two different geometric configurations, each with fixed regenerator positions, and observed consistent results. Combining them, they reported \( \Delta m = (0.542 \pm 0.006) \times 10^{10} \, \text{sec}^{-1} \).

At about the same time, Aronson et al. [25] reported a similarly high statistics measurement of mixing using two regenerators and varying the gap distance, the method introduced by Christenson et al. The experiment was performed at the zero-gradient synchrotron at the Argonne National Laboratory. To separate transmission events from diffraction events more cleanly, they chose a light target, \( \text{B}_4\text{C} \), for their primary measurement. To verify that the result was independent of regenerator material, they also made measurements using Cu regenerators. Combining measurements made with the \( \text{B}_4\text{C} \) and Cu regenerators, each in momentum ranges \( 2 < p < 3.0 \, \text{GeV/c} \) and \( 3.0 < p < 4.0 \, \text{GeV/c} \), they reported \( \Delta m = (0.542 \pm 0.006) \times 10^{10} \, \text{sec}^{-1} \). Not only was this central value the same as that reported by Cullen et al. [24], it was also the same as that of a weighted average, \( \Delta m = (0.542 \pm 0.013) \times 10^{10} \, \text{sec}^{-1} \), reported a year earlier [26], which included neither of these results.

Carnegie et al. did a similar measurement of \( \Delta m \) at the Alternating Gradient Synchrotron at Brookhaven National Laboratory as part of a program to measure the \( CP \)-violation parameters related to \( K_L^0 \to \pi^-\pi^+ \) precisely. They used Cu regenerators and varied the gap distance between 0 and 30 inches by placing the upstream regenerator on a motor-driven cart that completed a round-trip in 30 minutes. They reported \( \Delta m = (0.534 \pm 0.007) \times 10^{10} \, \text{sec}^{-1} \). They also reported the average of their result with those of Cullen et al. and Aronson et al. to be \( \Delta m = (0.539 \pm 0.0035) \times 10^{10} \, \text{sec}^{-1} \), a value with better than 1% precision.

In 1974, the CERN-Heidelberg-Dortmund group reported two independent measurements of \( \Delta m \) in back-to-back papers, each with precision better than the average reported by Carnegie et al. three years earlier. In the first analysis [27] they used a variant on the two regenerator method introduced by Christenson et al. [18] and the regenerator configuration introduced by Cullen et al. [24] where part of the beam was “sacrificed” to monitor the beam and the detector performance as a function of time. In the second [28] they extended an analysis of semileptonic decays observed in helium from which they had already extracted the \( \text{Re} \, \epsilon \) [29], the \( CP \) mixing amplitude in neutral kaon mixing. The measured values were

\[
\Delta m = (0.5340 \pm 0.0030) \times 10^{10} \, \text{sec}^{-1} \quad (24)
\]
\[
\Delta m = (0.5334 \pm 0.0040) \times 10^{10} \, \text{sec}^{-1} \quad (25)
\]

where the quoted errors include the statistical and systematic contributions added in quadrature. The
error on the average of these two measurements \((0.0024 \times 10^{10} \text{ sec}^{-1})\) is just under 0.5%.

Fermilab experiment E731 collected data to study \(CP\) violation and possible \(CPT\) violation during the 1987-1988 fixed target run. They published some of their results in a 1993 paper [30], including measurements of \(\Delta m\) from both \(\pi^-\pi^+\) and \(\pi^0\pi^0\) decays, denoted \((\Delta m)_{+-}\) and \((\Delta m)_{00}\), respectively. A single regenerator was used to produce the \(K^0_L\) beam whose amplitude interfered with the \(K^0_S\) amplitude to produce the \(\pi\pi\) final states. Where interference between \(K^0_S\) and \(K^0_L\) amplitudes introduced small correction factors into earlier measurements of \(\Delta m\), here it generated the oscillations in the decay rates which allowed \(\Delta m\) to be measured. As seen in a schematic diagram of the detector, Fig. 9, drift chambers and a dipole magnetic field were used to detect charged pions. A lead glass-glass calorimeter was used to detect electrons and photons. Its resolution was reported to be 1.5% \((2.5\%) + 5%/\sqrt{E}\) for electrons (photons), with average response understood to \(\pm 0.1\%\). Both the mass and primary vertex position in neutral events were reconstructed by pairing photons such that they were consistent with two \(\pi^0\)'s decaying from a common vertex. This allowed the kaon mass to be measured with average resolution 5.5 MeV in the neutral decay mode, to be compared to 3.5 MeV in the charged decay mode.

While the mass and vertex position resolutions were better for \(\pi^-\pi^+\) decays than for \(\pi^0\pi^0\) decays, the fiducial volume for neutral decays was significantly greater as charged decays had to occur before the upstream end of the drift chamber system. Fitting the exponential and interference terms of the decay rates to extract the \(K^0_S\) lifetime as well as the mass difference gave

\[
\begin{align*}
(\Delta m)_{+-} & = (0.5311 \pm 0.0044 \pm 0.0020) \times 10^{10} \hbar \text{ s}^{-1} \quad (27) \\
(\Delta m)_{00} & = (0.5274 \pm 0.0030 \pm 0.0017) \times 10^{10} \hbar \text{ s}^{-1} \quad (28) \\
\Delta m & = (0.5286 \pm 0.0028) \times 10^{10} \hbar \text{ s}^{-1} \quad (29)
\end{align*}
\]

where the statistical and systematic errors have been added in quadrature in the combined result. The projections of the fit results for the neutral decays are shown in Fig. 10.

Figure 9:

Essentially the same apparatus was used by the Fermilab E773 experiment to acquire data during the 1991 fixed target run. In conjunction with their tests of \(CPT\), they published a new measurement of \(\Delta m\) in 1995 [31]. Using only the \(K^0_S \rightarrow \pi^-\pi^+\) data, they reported

\[
\begin{align*}
\Delta m & = (0.5297 \pm 0.0030 \pm 0.0022) \times 10^{10} \hbar \text{ s}^{-1} \quad (31)
\end{align*}
\]

In this analysis they allowed the phase, \(\Phi_{+-} \equiv \text{arg}(\eta_{+-})\) to float in their fit rather than constraining
it to the superweak value

\[ \Phi_{+-} \approx \Phi_{00} \approx \phi_{SW} = \tan^{-1}[2\Delta m/(\Gamma_S - \Gamma_L)], \tag{33} \]

which had been done in the E771 analysis.

A measurement of \( \Delta m \) using a novel technique, with a similar statistical error and a significantly smaller systematic uncertainty, was reported later in 1995 by the CPLEAR experiment at CERN [32]. Antiprotons annihilated at rest in a high pressure hydrogen gas target to produce the channels \( K^+\pi^-K^0 \) and \( K^-\pi^+K^0 \). The charged kaons in the events tagged the initial flavors of the neutral kaons. The four decay rates, for each flavor to decay into each \( \pi e \nu \) charge state, were measured as functions of proper decay time, \( \tau \). The asymmetry between rates for unmixed and mixed decays, assuming \( \Delta S = -\Delta Q \), is proportional to \( \cos(\Delta m \tau) \). The fit to the asymmetry included a parameter to account for possible violation of the \( \Delta S = -\Delta Q \) rule. The result of the analysis was

\[ \Delta m = (0.5274 \pm 0.0029 \pm 0.0005) \times 10^{10} \text{ h s}^{-1}. \tag{34} \]

\[ \Delta m = (0.5295 \pm 0.0020 \pm 0.0003) \times 10^{10} \text{ h s}^{-1}. \tag{35} \]

In 1998 CPLEAR reported their final measurement of the mass difference extracted from semileptonic decays:

\[ \Delta m = (0.5295 \pm 0.0020 \pm 0.0003) \times 10^{10} \text{ h s}^{-1}. \tag{36} \]

\[ \Delta m = (0.5274 \pm 0.0029 \pm 0.0005) \times 10^{10} \text{ h s}^{-1}. \tag{37} \]

The KTeV Collaboration measured \( \Delta m \) as part of their study of direct CP violation and CPT symmetry. They collected data in 1996, 1997, and 1999. Their experimental configuration was conceptually similar to that of the earlier Fermilab experiments, E731 and E773. It used a single regenerator beam in parallel with a vacuum beam. They measured \( K \to \pi \pi \) rates sensitive to the interference of \( K_S^0 \) and \( K_L^0 \) amplitudes in the regenerator beam. While most of the decay rates could be attributed to the \( K_S^0 \) component of the beam, decays from the \( K_L^0 \) component and from \( K_S^0 - K_L^0 \) interference...
Table 1: $\Delta m$ and $\tau_S$ results for the regenerator beam charged and neutral data samples. The first uncertainty is statistical; the second is systematic.

<table>
<thead>
<tr>
<th>decay mode</th>
<th>$\Delta m \times 10^{10}$ h$^{-1}$</th>
<th>$\tau_S \times 10^{-12}$ s</th>
<th>$\chi^2$/dof</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi^+\pi^-$</td>
<td>$5266.7 \pm 5.9 \pm 14.3$</td>
<td>$89.650 \pm 0.028 \pm 0.074$</td>
<td>228/199</td>
</tr>
<tr>
<td>$\pi^0\pi^0$</td>
<td>$5237.3 \pm 10.6 \pm 18.3$</td>
<td>$89.637 \pm 0.050 \pm 0.126$</td>
<td>195/199</td>
</tr>
</tbody>
</table>

accounted for 20% of the decay rate, ranging from 5% near the regenerator to 90% at the end of the decay region. The interference term in the rate is proportional to

$$2 |\rho| |\eta| \cos(\Delta mt + \phi_p - \phi_\eta) e^{-t/\tau_{avg}}$$

where $\phi_\eta = \text{arg}(\eta)$, $|\rho|$ and $\phi_p$ are the magnitude and phase of the coherent regeneration amplitude, and $1/\tau_{avg} \equiv (1/\tau_S + 1/\tau_L)/2$. For the measurement of $\Delta m$ and $\tau_S$ the charged and neutral modes were fit separately assuming CPT symmetry so that the phases $\phi_{+}$ and $\phi_{00}$ were equal to superweak phase $\phi_{SW}$, see Eqn. (33). The data were fit in 12 momentum bins, each 10 GeV/c wide. An example of a decay length distribution, extracted from their 2003 paper [33] which reported results using the 1996 and 1995 data, is shown in Fig. 11. This also shows a Monte Carlo prediction without the interference term of Eqn. (38).

Figure 11: $z$ decay distribution of $K \to \pi^+\pi^-$ decays in the [KTeV] regenerator beam, for the restricted momentum range 40-50 GeV/c. The MC prediction (dashed) is without the interference term that is proportional to “$2 |\rho| |\eta|$” in Eq. 38. (Fig. 24 in Ref. [33])

The values of $\Delta m$ and $\tau_S$ extracted from the 1996 and 1997 $\pi^+\pi^-$ and $\pi^0\pi^0$ data are shown in Table 1. The difference between the charged and neutral mode results, after accounting for statistical and systematic uncertainties, is 1.6 $\sigma$ for $\Delta M$ and 0.1 $\sigma$ for $\tau_S$. In both modes the systematic uncertainties are greater than the statistical. Combining the charged and neutral mode results weighted by the statistical uncertainties and the independent parts of the systematic uncertainties,

$$\Delta m = (5261 \pm 15) \times 10^{10} \text{ h}^{-1}$$
$$\tau_S = (89.65 \pm 0.07) \times 10^{-12} \text{ s}.$$  

Very recently, KTeV has reported an updated analysis using all of the data (about twice that analyzed earlier) and using an improved Monte Carlo simulation of the experiment [34]. This allows improved systematic uncertainties as well as reduced statistical uncertainties. Imposing CPT invariance a priori, as in [33], they now report

$$\Delta m = (5269.9 \pm 12.3) \times 10^{10} \text{ h}^{-1}$$
$$\tau_S = (89.623 \pm 0.047) \times 10^{-12} \text{ s}.$$
In addition, they perform a fit allowing CPT violation for which they report

$$\Delta m = (5279.7 \pm 19.5) \times 10^{10} \text{ h} \text{s}^{-1}$$  \hspace{1cm} (43)

$$\tau_S = (89.589 \pm 0.070) \times 10^{-12} \text{s}.$$  \hspace{1cm} (44)

3 Mixing in the $D^0$-$\overline{D}^0$ System

$D^0$-$\overline{D}^0$ mixing was described even before charm was discovered. Gaillard, Lee, and Rosner wrote a prescient paper on the “Search for Charm” [35], published in January 1975. It discussed the phenomenology expected for mesons containing a then-hypothetical fourth quark. It noted that both $\Delta \Gamma$ and $\Delta m$ would be small compared to the lifetimes of the weak eigenstates due to the Cabibbo-suppression of transitions connecting the particle and antiparticle states, concluding the discussion by noting that “the effects of $D^0$-$\overline{D}^0$ mixing are not important in the decays of these particles.” Shortly after the paper had been written, while it was in proof, the $J/\psi$ meson and various $\psi'$ mesons were discovered, initiating the November Revolution of particle physics. The $D^0$ was discovered less than 2 years later [36], with the anticipated properties.

The first observation of the decay $D^{*+} \rightarrow D^0 \pi^+$ was reported in 1977 [37]. The first limit on $D^0$-$\overline{D}^0$ mixing was also reported in that paper. Data were collected from 160,000 hadronic interactions in the $E_{c.m.}$ range 5.0 to 7.3 GeV with the MARK-I detector at SPEAR. Using time-of-flight, each particle in a multihadronic event was assigned a weight for each of the $\pi/K/p$ hypotheses. Each $K^\mp \pi^\pm$ candidate with momentum $> 1.5$ GeV/c was weighted using the product of the daughter particles’ weights. A $D^0$ peak with $87 \pm 25$ weighted combinations corresponding to about 250 events was clearly visible in the data. The observed width, 34 MeV, was consistent with detector resolution. $K^-\pi^+$ (and charge conjugate) combinations in the mass range 1820 to 1910 MeV/$c^2$ were combined.
with an additional pion and $D\pi - D$ invariant mass spectra plotted. The results, with no weighting for particle identification, are shown in Fig. 12.

The upper plot shows a clear peak in the channel $D^0\pi^+$ with the $D^0$ decaying to the Cabibbo-favored final state $K^-\pi^+$. The lower plot shows at most a small signal in the channel $D^0\pi^-$ with $D^0 \to K^-\pi^+$. While the focus of the paper was determining $D^{*+}$ characteristics from the signal observed in the upper plot, the authors also discussed possible contributions to the signal region in the lower plot. They considered double particle misidentification of $D^0$ daughters, doubly Cabibbo-suppressed decays ($\Delta C = -\Delta S$) of the $D^0$, and $D^0 - \bar{D}^0$ mixing, a possibility which had already received wide speculation in the literature (see ref. 5 in the paper). They derived a 90% confidence level upper limit on the fraction of the time a $D^0$ decays as if it were a $\bar{D}^0$ to be 16%.

Many subsequent searches and studies of mixing in the charm sector use the key techniques introduced in this seminal analysis. First, the pion from charged $D^*$ decay is used to tag the flavor of the daughter neutral meson when it is born. This is often referred to as the slow pion, $\pi_s$. Because the $Q$-value of the $D^*$ decay is small, the $\pi_s$ momentum is small relative to that of the $D^0$ decay products if the $D^*$ is produced at rest or the $\pi_s$ momentum is typically a factor of 5 to 10 less than that of the $D^0$ if the $D^*$ is produced with large lab momentum. Second, the mass difference between the $D^*$ candidate and the $D$ candidate is used to identify a signal region. We will use $\Delta M$ to denote this mass difference as we already use $\Delta m$ to denote the difference of mass eigenstates, even though the latter is commonly used in the literature. Third, mixing candidates are distinguished from dominant decays by the charge of $\pi_s$ relative to the charge of the $D$’s daughter kaon. Cabibbo-favored (CF) $D$ decays produce kaons with charge opposite that of the $\pi_s$ while doubly Cabibbo-suppressed (DCS) decays and CF decays following mixing produce kaons with charge equal that of the $\pi_s$. For more than a decade after the discovery of charm, experiments struggled to produce large, clean $D$ signals, and one method to reduce backgrounds in $D^*$ decays was to separate the data into what were called right-sign (RS) samples corresponding to CF decays and wrong-sign (WS) for everything else. Although somewhat misleading - there is nothing wrong with DCS or mixed decays - this nomenclature remains in use.

The MARK-I collaboration published a second paper with a comparable mixing limit using a different technique a few months later [38]. Roughly equal samples of $e^+e^-$ annihilation data were collected at fixed energies $E_{c.m.}$ = 4.028 GeV and 4.415 GeV. Fully reconstructing $D^0 \to K^-\pi^+$ candidates, they studied the recoil spectra and determined that most events were produced as $D\bar{D}^*$ or $D^*\bar{D}^*$. From the events with kaons identified in the recoil system as well as in the $D^0 \to K\pi$, they concluded that $(12 \pm 9)\%$ had same-sign kaons. This fraction was consistent with the 13% obtained from a Monte Carlo simulation predicated on no mixing. From this, they concluded that less than 18% of events containing a $D^0$ exhibit apparent strangeness violation, and hence mixing, at the 90% confidence level.

A somewhat similar technique was used by a Fermilab experiment E595, a fixed target hadroproduction experiment, to provide a more stringent limit which was published in 1981 [39]. They measured the rates of events with same-sign and opposite-sign dimuon events with missing energy indicative of final state neutrinos from charm (or beauty) decay. Semileptonic $D^0$ ($\bar{D}^0$) decay always produces positively (negatively) charged leptons. There is no leptonic equivalent of the WS kaons produced in singly or doubly Cabibbo-suppressed decays. More than 90% of $\mu^+\mu^-$ events observed in 278 GeV (350 GeV) $\pi^-$-Fe (proton-Fe) interactions with missing energy greater than 40 (45) GeV were found to originate from charm particle decays. After subtracting background and combining the data samples, they reported a 90% CL upper limit of 1.8% for the fraction of prompt same-sign dimuon events with large missing energy. Making conservative assumptions about the production and semileptonic decay of charm (which were poorly constrained at the time), they quoted an upper limit of 4.4% for the fraction of $D^0$ semileptonic decays that result in a wrong-sign muon.

Another variation on this theme was used by team working at CERN. In 1985 they reported results from a study of so-called wrong-sign dimuon events in deep inelastic scattering of 200 GeV muons
from a carbon target [40]. They compared the number of $\mu^+N \rightarrow \mu^+\mu^-\mu^-X$ and $\mu^-N \rightarrow \mu^-\mu^+\mu^+X$ events to the number of $\mu^+N \rightarrow \mu^+\mu^+\mu^-X$ events. The experiment was performed at the CERN SPS muon beam. The apparatus was a 50 m long toroidal iron magnet, centered on the beam axis and instrumented with scintillation counters and multiwire proportional chambers to detect the scattered muons. The central bore of the magnet contained a 40 m long carbon target. The total number of incoming muons was $1.75 \times 10^{12}$, and the number of -wrong-sign events in the fiducial volume was 17. The estimated number of background events due to double hadron decay-in-flight was $15.5 \pm 2.8$, clearly consistent with the observed number of signal candidates. To determine a mixing limit, $\sigma(c\bar{c})$ production cross sections were estimated from a variety of sources, as well as fragmentation and decay properties of charmed hadrons, to determine an effective cross section for charm production where only 0 background events were expected and 162 $S = 0$ observed [46]. They searched for

$$e^+e^- \rightarrow \psi'' \rightarrow D^0\overline{D}^0$$

(45)

with both neutral $D$’s fully reconstructed within the detector. The events were fit kinematically to the hypothesis that two particles of mass $M$ (not constrained) were produced and subsequently decayed into one of the three modes $K\pi$, $K\pi\pi^0$, $K\pi\pi\pi$ (where $K$ and $\pi$ with no superscript denote charged mesons). The particle identification system combined information from time-of-flight counters and $dE/dx$ in the drift chamber to distinguish kaons from pions. The apparent $S = \pm 2$ rate was much greater than the expected from DCS decays. It was also much greater than expected from mixing due to conventional physics, so was considered a possible signature for new and exciting physics. In particular, one event was a candidate for the final state

$$e^+e^- \rightarrow D^0\overline{D}^0 \rightarrow (K^-\pi^+), (K^-\pi^+)$$

(46)

which Bose symmetry forbids for DCS decays but allows with mixing.

In 1986 Fermilab experiment E615 reported a limit on $D^0 - \overline{D}^0$ mixing from a study of high-mass dimuons produced in 255 GeV $\pi N$ interactions [47]. The principal goal of the experiment was to explore the structure of the pion by measurement of $\mu^+\mu^-$ pairs with mass greater than 4 GeV/c$^2$ produced by $q\bar{q}$ annihilation. They used lower mass $\mu^+\mu^-$ pairs to search for $D^0 \rightarrow \mu^+\mu^-$ and analyzed same-sign $\mu^\pm\mu^\pm$ pairs to set limits on mixing where the production of a $D^0 \overline{D}^0$ pair was followed by semileptonic decays of both mesons. The pion beam interacted in a tungsten target upstream of a magnet whose magnetic volume was filled with low-Z material to absorb secondary hadrons. A downstream spectrometer consisting of sixteen planes of drift chambers, nine planes of multiwire proportional, and four scintillator hodoscopes surrounded the analysis magnet. Candidate track pairs were required to penetrate an additional 2 m of iron at the rear of the apparatus to confirm that they were muons. A hardware processor used hodoscope information to trigger only on muon pairs with invariant mass above 2.0 GeV/c$^2$.

To search for mixing they found it most effective to examine the $\cos \theta$ distribution of the same-sign pairs, where $\theta$ was defined to be the angle between the direction of the incident pion and a $\mu$ in the muon-pair rest frame. The data and a histogram from a “random-pair sample” are shown in Fig. 13. The latter was constructed by randomly combining a muon from a prescaled-trigger sample
and combining it with another such muon. The agreement between the directly observed data and the random-pair sample suggests that the like-sign muon pair were essentially all due to uncorrelated background. Using Monte Carlo simulations for a variety of production models, two shown in the figure, they conservatively reported an upper limit on the mixing rate of 0.56% at the 90

In 1988 Fermilab experiment E691 reported a search for mixing in a photoproduction experiment at the Tagged Photon Laboratory (TPL) [48]. Building on technology developed at CERN, this was the first HEP experiment to successfully employ a silicon microstrip vertex detector [49] to measure charm decay times precisely [50, 51]. The experiment used the very good decay time resolution in two similar, but complementary, ways. First, charm signals were separated from background by requiring a candidate’s decay time, $t_d > 0.22$ ps, about half a $D^0$ lifetime. This produced a very clean sample of approximately 1000 $D^* \to D^0 \pi^+$ decays followed by RS $D^0 \to K^- \pi^+$ or $K^- \pi^- \pi^+ \pi^+$ (or charge conjugate). Second, WS decays (signalling mixing or DCS decays) were searched for in the sample with $t_d > 0.88$ ps. In the limit $|x|, |y| \ll 1$, and ignoring possible CP violation, the WS decay rate, Eqn. (11), becomes

$$|\mathcal{M}|^2 = e^{-\Gamma t} |\mathcal{A}_\alpha|^2 \left( R_D^2 + \sqrt{R_D} y' (\Gamma t) + \frac{x^2 + y^2}{4} (\Gamma t)^2 \right)$$

where $\sqrt{R_D} = |\overline{\mathcal{A}}_\alpha|/|\mathcal{A}_\alpha|$ is the ratio of the magnitudes of the DCS and CF amplitudes to their common final state and $y' = y \cos \delta - x \sin \delta$ ($x' = x \cos \delta + y \sin \delta$) depends on the mixing parameters and the relative strong phase $\delta$ between $\mathcal{A}_\alpha$ and $\overline{\mathcal{A}}_\alpha$. DCS decays had not yet been observed, and the higher lifetime cut on the WS decays especially increased sensitivity to pure mixing, the quadratic term. Ignoring the interference term, 90% CL upper limits for the mixing rates in the $K\pi$ and $K\pi\pi\pi$ samples were reported to be 0.50% and 0.48%. A combined limit of 0.37% was reported, along with limits for the DCS rates, relative to CF rates, of 1.5% and 1.8%, well above those expected. The effects of interference between the DCS and mixing amplitudes were explicitly considered, and reasons why
a scenario with maximal destructive interference was unlikely were presented. While E791 produced mixing limits only marginally stronger than those from Ref. [47], it introduced the use of silicon microstrip detectors and the time dependence of amplitudes to searches for charm mixing.

Figure 14: Figure 1 in [52]

The CLEO Collaboration reported limits for WS $D^0 \to K\pi$ and $K\pi\pi\pi$ decays in 1992 [53] and and the CLEO-II Collaboration reported evidence for the WS $D^0 \to K\pi$ decay in 1994 [52]. Both analyses used data collected in $e^+e^-$ interactions in the $\Upsilon$ energy range at the Cornell Electron Storage Ring. The CLEO-II experiment exploited both $dE/dx$ and time-of-flight information to distinguish pions from kaons. As previous experiments had, it used the $D^{*+} \to D^0\pi^+$ (and charge conjugate) decay chain to search for RS and WS signals. Candidates were required to have large momentum fraction in the center-of-mass to reduce combinatorial backgrounds. The $\Delta m$ distributions for RS and WS candidates with $K\pi$ mass within 22 MeV/$c^2$ of the $D^0$ mass (about 2 standard deviations) is shown in Fig. 14. The numbers of RS and WS events were estimated to be $2465 \pm 50$ and $19.1 \pm 6.1$, respectively, from which the WS/RS ratio was determined to be

$$R = 0.0077 \pm 0.0025 \text{ (stat)} \pm 0.0025 \text{ (syst)}.$$  \hbox{(48)}

This corresponds to

$$[2.92 \pm 0.95 \text{ (stat)} \pm 0.95 \text{ (syst)}] \tan^4 \theta_C$$  \hbox{(49)}

where $\theta_C$ is the Cabibbo angle. The detector did not measure decay times, so the experiment reported that the signal could be due to mixing, DCS decays, or a combination of both. In retrospect, the central value reported for the WS/RS ratio is roughly consistent with the DCS value which is close to $1.5 \tan^4 \theta_C$.

Fermilab experiment E791 recorded data from approximately $2 \times 10^{10}$ hadronic interactions in the 1991-1992 fixed-target run using the TPL spectrometer with a 500 GeV/$c\pi^-$ beam incident on 5 thin foil targets. It published mixing results in a series of papers between 1996 and 1999 [54, 55, 56]. Beam
tracks and interaction products were measured using both silicon strip and wire chamber detectors. Critically, primary vertices were measured with 350 \( \mu \text{m} \) longitudinal and 6 \( \mu \text{m} \) transverse resolution while \( D^0 \) hadronic decay vertices were measured with \( O(350 \mu \text{m}) \) resolution at a characteristic momentum of 65 GeV/\( c \). This provided almost 10\( \sigma \) separation at one lifetime and 0.1 lifetime decay time resolution. By selecting \( D^0 \) candidates with decay vertices well-separated from (and pointing back to) primary vertices, clean RS samples were measured and WS samples studied for evidence of mixing and DCS signals.

In the first paper published \([54]\), E691 studied \( D^{*+} \rightarrow D^0 \pi^+; \ D^0 \rightarrow K\ell\nu \ (\pm cc) \) candidates. Because the primary and secondary vertices were required to be well-separated, at least 8 standard deviations, using a constrained fit allowed an estimate of the neutrino momentum, up to a twofold ambiguity. The solution resulting in the higher \( D^0 \) momentum was used for all events. Using this momentum along with those directly measured, \( \Delta m \) was estimated for each candidate. The resulting RS and WS distributions, integrated over \( D^0 \) decay time, for semielectronic and semimuonic samples are shown in Fig. 15. To search for mixing, separate unbinned fits were performed on the Ke\( \nu \) and K\( \mu \nu \) samples using \( \Delta m \) and proper decay time for each event. As there are no DCS semileptonic decays, a WS signal would be due to mixing and its decay time distribution would be proportional to \( t^2 e^{-\Gamma t} \) while the RS decay time distribution is proportional to \( e^{-\Gamma t} \), modulo detector efficiency. The fits found 1237 \( \pm 45 \) RS and \( 4.1^{+11.8}_{-10.8} \) WS Ke\( \nu \) events and 1267 \( \pm 44 \) RS and \( 1.8^{+12.1}_{-11.0} \) WS K\( \mu \nu \) events. Accounting for efficiency as a function of decay time, the mixing rate was reported to be \( r = (0.11^{+0.30}_{-0.27})\% \) with a 90\% CL upper limit \( r < 0.50\% \). Although this limit was slightly weaker than that reported earlier by E691, it was less reliant on assumptions related to possible interference between DCS and mixing amplitudes.

E791 also searched for mixing and DCS signals using K\( \pi \) and K\( \pi\pi\pi \) final states. The paper reporting these results \([55]\) was published as a traditional, full-length article in Physical Review D, so includes many details of the experiment relevant for all three of their papers discussed here. Signals
The integrated S:B ratio in the D in the final sample was approximately 3200 (35 400) and the average weight was approximately 2.08 of the CP-even eigenstate while the D and decay rates, the eigenstates are the physical eigenstates of the neutral charm meson system, with well-defined masses by comparing the measured lifetimes of different CP eigenstates. In the limit that the CP shop [58] that one could measure the difference of the decay rates (inverse lifetimes) of the physical errors larger than the value itself.

K to account for its particle-identification efficiency. The number of unweighted D significance of the target foils. Particle identification from two Cherenkov detectors was used to improve the statistical σ To improve the cleanliness of the samples, decay vertices were required to lie at least 4 outside the decay time distribution is exactly exponential with the lifetime the average of those of the CP-even and CP-odd eigenstates. The central results are shown in Table 3. which is extracted from the paper. Depending on the assumptions made about CP violation in mixing and DCS amplitudes, the quoted 90% CL upper limits for r_mixed ranged from 0.74% to 1.45%. The importance of allowing for CP violation were mixing close to the experimental bound had not been considered in previous analyses, and had only recently been identified as an important phenomenological issue [57]. Assuming no mixing, the central value for the D0 → Kπ DCS rate was close to the anomalously high value previously reported by CLEO [52] and the D0 → Kπππ DCS rate was very close to that theoretically expected, but with statistical errors larger than the value itself.

The third mixing analysis reported by E791 [56] followed a suggestion at the CHARM2000 workshop [58] that one could measure the difference of the decay rates (inverse lifetimes) of the physical eigenstates by comparing the measured lifetimes of different CP eigenstates. In the limit that the CP eigenstates are the physical eigenstates of the neutral charm meson system, with well-defined masses and decay rates, the D0 → K−K+ decay time distribution is exactly exponential with the lifetime of the CP-even eigenstate while the D0 → K−π+ decay time distribution is, to a very good approximation, exponential and its lifetime the average of those of the CP-even and CP-odd eigenstates. To improve the cleanliness of the samples, decay vertices were required to lie at least 4 σ outside the target foils. Particle identification from two Cherenkov detectors was used to improve the statistical significance of the D0 → K−K+ sample, and similar particle identification criteria were applied to the D0 → K−π+ sample to minimize systematic uncertainties. Each two-track combination was weighted to account for its particle-identification efficiency. The number of unweighted K−K+ (K−π+) events in the final sample was approximately 3200 (35 400) and the average weight was approximately 2.08 (1.71). The integrated S:B ratio in the D0 → K−K+ signal region (±2.5σ) was ≈ 1 : 1 and the decay
Figure 16: Distributions and the exponential fits for the number of $D^0 \rightarrow K^-\pi^+$ (top) and $K^-K^+$ (bottom) decays as a function of reduced proper decay time, after particle identification weighting and acceptance corrections. (Figure 3 in [56])

time distribution of background events estimated from mass sidebands. The integrated S:B ratio in the $D^0 \rightarrow K^-\pi^+$ signal region was $\approx 3 : 1$. The reduced time distributions (decay time measured relative to that required to survive selection criteria) for the weighted, background subtracted, and efficiency-corrected samples are shown in Fig. ???. The lifetimes were measured to be $(0.413 \pm 0.003 \pm 0.004)$ ps for the $K^-\pi^+$ sample and $(0.410 \pm 0.011 \pm 0.006)$ ps for the $K^-K^+$ sample. The difference in decay widths was calculated to be

$$\Delta \Gamma = (0.04 \pm 0.14 \pm 0.05) \text{ ps}^{-1}$$

and a 90% CL interval determined to be

$$-0.20 \text{ ps}^{-1} < \Delta \Gamma < 0.28 \text{ ps}^{-1}.$$  

Add a statement of the range for $y$. Also an statement that the central values are consistent with currently accepted values.

A similar analysis was reported in 2000 by the Wideband photoproduction experiment FOCUS [59] using recorded data during the 1996-1997 fixed target run at Fermilab. The photon beam was derived from the bremsstrahlung of secondary electrons and positrons with an $\approx 300$ GeV endpoint energy produced by 800 GeV/c protons. The spectrometer included two analysis magnets of opposite polarity, silicon microstrip detectors which provided average proper decay time resolution $\approx 30$ fs for 2-track charm decays, multiwire proportional chambers for measuring momenta, and three multicell threshold Cerenkov detectors to discriminate between electrons, pions, kaons, and protons. $D^0 \rightarrow K^-K^+$ and $D^0 \rightarrow K^-\pi^+$ candidates entered the analysis samples via both tagged and inclusive paths. Both required vertex separation greater than 5 $\sigma$. The former required that the $D^0$ be a $D^{*+}$ decay candidate with $\Delta m$ within 3 MeV of the nominal value while the latter employed more stringent
Figure 17: Signal versus reduced proper time for $D^0 \rightarrow K^-\pi^+$ and $K^-K^+$ requiring $W_\pi - W_K > 4$ and $\ell/\sigma > 5$. The fit is over 20 bins of 200 fs bin width. The data is background subtracted and includes the (very small) Monte Carlo correction. (Figure 3 in [59])

kinematic and Cherenkov cuts to reduce backgrounds in the signal region and also reflections in the $D^0$ mass sidebands. The analysis was designed to minimize relative systematic errors rather than obtain the best statistical errors on the lifetimes. Candidates from the tagged and inclusive channels were combined for the lifetime fits, explicitly assuming CP symmetry. The S:B ratios were similar to those of E791, but the size of the samples were about three times larger. Background decay time distributions in the signal regions were estimated from decay time distributions in sidebands. The reduced proper decay time distributions for signal, shown in 17, were fit for the difference in lifetimes and the $D^0 \rightarrow K^-\pi^+$ lifetime, allowing the background levels in each time bin to float, with some constraints imposed. The fit results were

$$y_{CP} = (3.42 \pm 1.39 \pm 0.74)\%$$ \hspace{1cm} (52)

and

$$\tau(D^0 \rightarrow K^-\pi^+) = (409.2 \pm 1.3) \text{ fs}$$ \hspace{1cm} (53)

where the reported error was statistical only. From these, they computed

$$\tau(D^0 \rightarrow K^-K^+) = (395.7 \pm 5.5) \text{ fs},$$ \hspace{1cm} (54)

again with statistical error only. A large number of cut variants and fit variants were considered. In the authors' conclusions, they said, “[b]ecause of our high statistics, the error on $y_{CP}$ reported here can be reliably interpreted as a Gaussian error for the purposes of combining with other measurements”.

Also in 2000, the CLEO II.V experiment at the Cornell Electron Storage Ring (CESR) reported results from a study of WS $D^0 \rightarrow K\pi$ events produced in 9.0 fb$^{-1}$ of $e^+e^-$ collisions at $\sqrt{s}$ 10 GeV [61]. The data were accumulated between February 1996 and February 1999. The CLEO II detector
Figure 18: Allowed regions, at 95% C.L., in the $y'$ vs. $x'$ plane. The entire kidney shaped region, filled with tight cross-hatching, is allowed under Fit A of Table II, while Fit B, in which CP violation is assumed, allows the smaller region, which is overlayed and filled with looser cross-hatching. The allowed regions from studies comparable to Fit A, using $D^0 \rightarrow K^+ K^-$ [56], for which we assume $\delta = 0$, and $D^0 \rightarrow K^+ \ell^- \bar{\nu}_\ell$ [54], are shown as single-hatched regions. The Bayesian approach is used [60]. (Figure 3 in [61])

had been upgraded with the addition of a silicon vertex detector (SVX) and a change in drift chamber gas from argon-ethane to helium-propane. CLEO II.V tagged the flavor of neutral charm mesons at production using slow pions from charged $D^*$ decay. Measuring the $D^0$ decay vertex and flight direction precisely, they improved the $\Delta m$ resolution by a factor of 3 by using the intersection of the $D^0$ candidate’s momentum with the beam spot as an additional point on the slow pion track [62]. This innovation greatly reduced the direction error resulting in the slow pion’s direction from its Coulomb multiple scattering in the beampipe and detector materials. As as it improved the $S : B$ ratio in the WS sample by almost as large a factor as it improved the $\Delta m$ resolution, it improved both the statistical and systematic uncertainties dramatically. The intersection of the beams produced a luminous region $7 \mu$m high $\times$ $700 \mu$m wide $\times$ 30,000 $\mu$m long. The typical three-dimensional flight distance of a $D^0$ was 180 $\mu$m and the typical resolution in each direction 40 $\mu$m. The $D$ decay time was reconstructed using only the vertical component of $D$ flight distance, giving a typical decay time precision 0.4 $D^0$ lifetimes. The estimated number of WS signal events was $44.8^{+9.7}_{-8.7}$ with a similar number of background events in the signal region. Fitting the time-dependence of the WS events using the generalization of Eqn. 47 allowing the most general CP violation, they found $R_D = (0.48 \pm 0.12 \pm 0.04)\%$ and
values for the mixing parameters consistent with zero. Setting the mixing terms to zero, they found
\[ R_D = (0.332^{+0.063}_{-0.065} \pm 0.040)\% . \]
The limits for the mixing parameters are shown in Fig. 18, extracted from [61], which compares them with earlier limits from E791 [54, 56]. In making the comparison of \( y' \) from this experiment with \( y \) from E791, the relative strong phase between the CF and DCS \( K\pi \) amplitudes is explicitly assumed to be \( \delta = 0 \).

In 2001 the FOCUS experiment reported the ratio of WS to RS \( K\pi \) events from charged \( D^* \) candidates to be \( (0.404 \pm 0.085 \pm 0.025)\% \) [63] with the assumption of no mixing and no CP violation. They also considered the variation in \( r_{des} \) as a function of \( y' \) for two values of \( x' \) covering the 95% CL of the CLEO II V result [61].

CLEO reported the first observation of WS \( D^0 \rightarrow K^+\pi^-\pi^0 \) decays [64] and reported the WS/RS rate, integrated over decay time, to be \( (0.43^{+0.11}_{-0.10} \pm 0.07)\% \). The data reconstruction and candidate selection criteria roughly paralleled those they used in their WS \( K^+\pi^- \) analysis [61], but they did not attempt an analysis of the proper time distribution. They noted that doing so "in this mode is more difficult than in the \( K^+\pi^- \) mode". Effectively, the values of \( x' \) and \( y' \) vary as functions of the position in the Dalitz plot, so extracting a set of mixing parameters requires a time-dependent amplitude analysis, as will be discussed in when the results of such analyses are presented. Similarly, they reported the time-integrated ratio of WS \( D^0 \rightarrow K^+\pi^-\pi^* + \pi^+ \) to RS \( D^0 \rightarrow K^-\pi^-\pi^* + \pi^+ \) rates to be \( R_{WS} = \left( (0.42^{+0.12}_{-0.11} \pm 0.04) \times 10^{-2} \right) \times (1.07 \pm 0.10) \) [65] where the multiplicative factor relates to possible differences in reconstruction efficiencies due to the WS and RS signals populating the five-dimensional decay phase space differently.

The Belle experiment at KEK, an asymmetric \( e^+e^- \) experiment operating near the \( \Upsilon(4S) \) and designed to produce boosted \( B \) mesons, reported a search for the difference between \( D^0 \rightarrow K^-\pi^+ \) and \( D^0 \rightarrow K^-K^+ \) lifetimes from a 25 fb\(^{-1} \) sample in 2002 [66]. The detector had a 3 layer silicon vertex detector (SVD) (later upgraded), a 50 layer central drift chamber (CDC), a system of aerogel Cherenkov counters (ACC), and time-of-flight (TOF) scintillation counters which were critical for the analysis. Particles were identified using de/dx information from the CDC in conjunction with information from the ACC and TDC systems. \( D^0 \) candidates were required to point back to the interaction point. There were \( 214 260 \pm 562 K^-\pi^+ \) and \( 18306 \pm 189 K^-K^+ \) signal events with 87% and 67% purity, respectively, within 3 \( \sigma \) of the \( D^0 \) mass. A maximum likelihood fit was used to extract \( \tau(K^-\pi^+) = 416.2 \pm 1.1 \) fs and \( y_{CP} = (-0.2 \pm 1.0)\% \). After corrections for bias determined from Monte Carlo simulations, and studies of systematic uncertainties in the differences of the measured lifetimes, they reported
\[ y_{CP} = (-0.5 \pm 1.0^{+0.7}_{-0.8}) \times 10^{-2} \] (55)
and a corresponding 95% confidence interval (CI) \(-0.030 < y_{CP} < 0.020 \). They noted that their result was consistent with zero, alluding to the fact that the CI excluded the 3.42% central value reported earlier by FOCUS [59]. CLEO published a less precise and consistent result from a 9 fb\(^{-1} \) sample,
\[ y_{CP} = (-1.2 \pm 2.5 \pm 1.4) \times 10^{-2} , \] (56)
the same month [67].

The \( \text{BaBar} \) detector at SLAC’s PEP-II asymmetric \( e^+e^- \) collider was very similar to the Belle detector at KEK. It also recorded data near the \( \Upsilon(4S) \) resonance with a primary focus on measuring CP violation in \( B \) decays. Rather than a aerogel threshold Cherenkov detector, it had a ring-imaging Detector of Internally Reflected Cherenkov light (DIRC) for particle identification and it had a 5-layer silicon vertex tracker from the beginning. In 2003 it published a study of WS \( K\pi \) events from a 57 fb\(^{-1} \) sample [68]. The general approach was very similar to that of the earlier CLEO analysis [61], but the mixing limits were determined determined in \((x'^2, y')\) space using a purely frequentist approach.

The WS to RS ratio as a function of decay time is approximated as a quadratic relationship in the limit \( x, y \ll 1 \). Physically, the linear term may be positive or negative, but the quadratic term must
be positive, see Eqn. 47. Fits to data can, of course, produce negative coefficients for the quadratic term. This corresponds to negative values of $x'^2$. CL contours extending into unphysical regions in $(x', y')$ are expected due to statistical fluctuations, just as negative values for branching fractions for rare decays are expected in the absence of signal, and this is not a problem. However, extreme unphysical values for parameters can predict negative numbers of WS events (not just mixing events) for some decay times, and this can lead to undefined likelihoods in maximum likelihood fits, so must be addressed carefully. The results of BaBar’s fit, including systematic uncertainties and shown for cases assuming CP conservation and allowing for CP violation are shown if Fig. 19. The most likely point assuming CP conservation is the the unphysical region. The contours of the allowed region indicate a negative correlation between the allowed values of $x'^2$ and $y'$: to the extent that a larger value of $x'^2$ increases the WS/RS ratio at high decay times, a lower value of $y'$ tends to decrease it. This orientation of the error “ellipse” is characteristic of the quadratic approximation and will be seen in all similar analyses. In addition to the results of mixing fits, the paper reported the ratio of DCS to CF $K\pi$ decays to be $R_D = (0.357 \pm 0.022 \pm 0.027)$% in the limit of no mixing.

In 2004 and 2005 the usual suspects reported a series of new limits using larger data sets and somewhat more sophisticated versions of analyses similar to those done earlier. Babar studied semileptonic decays in 87 fb$^{-1}$ of data and reported $R_{\text{mix}} < 0.0042 (0.0046)$ at the 90% (95%) CL [70]. Belle studied mixing using WS $K\pi$ decays from 90 fb$^{-1}$ of data [69] and determined the $(x'^2, y')$ limits shown in Fig. 20. FOCUS reanalyzed the same data set considered earlier [59] and reported similar mixing limits using a different analysis technique [71]. CLEO searched for mixing in semileptonic decays using 9 fb$^{-1}$ of data and reported $R_{\text{mix}} < 0.0078$ at 90% CL [72]. Belle searched for mixing in semileptonic decays using 253 fb$^{-1}$ of data and reported $R_{\text{mix}} < 1.0 \times 10^{-3}$ at 90% CL [73].

The CLEO collaboration published a seminal paper using $D^0 \rightarrow K_S^0 \pi^- \pi^+$ in 2005 [74]. Although the experiment’s relatively low statistics limited its sensitivity, this was the first time a time-dependent amplitude analysis was used to study charm mixing. Such analyses are generically powerful, and the $K_S^0 \pi^- \pi^+$ final state has contributing amplitudes with well-defined CP symmetry. Such intermediate amplitudes allow $x$ and $y$ to be determined without complications from unknown relative strong phases between direct and mixed amplitudes, $A_\alpha$ and $\bar{A}_\alpha$, leading to the same final state. The full mixing

![Figure 19: 95% CL limits in $(x'^2, y')$ with and without CP violation (CPV) allowed. The solid point represents the most likely fit point assuming CP conservation and the open circle the same but allowing CP violation and forcing $x'^2 > 0$. The dotted (dashed) line is the statistical (statistical and systematic) contour for the case where no CP violation is allowed. The solid and dash-dotted lines are for the corresponding case where CP violation is allowed. (Figure 3 in [68])](image-url)
formalism for a neutral $D$ with well-defined flavor at $t = 0$ is given in Eqn. (11). In the limit $x, y \ll 1$, it takes the form given in Eqn. 47. For a three-body decay $A_\alpha$, $R_D$ and $y'$ are functions of position in the Dalitz plot. In addition to the mixing parameters, the magnitudes and phases of intermediate amplitudes are determined from the data. The relative strong phases of CF and DCS amplitudes with well-defined CP symmetry, such as $D \to K_S^0 \rho^0$, are constrained according to the CP eigenvalue, and the relative phases of the other contributing amplitudes are allowed to float in the fits.

The following year, BaBar reported a search for mixing and a branching ratio measurement in WS $D^0 \to K^+\pi^-\pi^0$ decays [75]. Rather than doing a time-dependent amplitude analysis over the full Dalitz plot, the search for mixing was restricted to regions of phase space where the mixing signal was expected to be enhanced. The data were fit to the form of Eqn. 47 with the coefficients of each term interpreted as quantities that had been integrated over the choice of phase-space regions. The mixing rate was reported to be $R_M = (0.023^{+0.018}_{-0.014} \pm 0.004)\%$ and $R_M < 0.054\%$ at the 95% CL, assuming CP invariance. The data were consistent with no mixing at the 4.5% confidence level.

The CDF experiment at Fermilab’s Tevatron entered the game of WS charm measurements in 2006. They reported the ratio of WS/RS signal events to be $R_B = (4.05 \pm 0.21 \pm 0.11) \times 10^{-3}$, assuming no time-dependence (negligible mixing) [76]. Their data are illustrated in Fig. 21. Events were selected in real time requiring, amongst other criteria, that the two $D^0$ daughter track candidates satisfied transverse impact parameter cuts consistent with finite decay time with respect to the primary vertex position. They also noted that, “[u]sing the technique we have established to extract the $D^0 \to K^+\pi^-$ signal, we can perform a time-dependent analysis using a larger data sample than reported here, to separately measure $R_D$ and the mixing parameters $x'$ and $y'$.”

The Belle collaboration also updated their WS/RS $K\pi$ analysis from 90 fb$^{-1}$ of data to 400 fb$^{-1}$ in 2006 [77]. They fit the data under three hypotheses: mixing with no CPV, mixing allowing full CPV, and no mixing. In the limit of no mixing, they reported $R_D = (3.77 \pm 0.08 \pm 0.05) \times 10^{-3}$. From a fit for mixing with no CPV, they found $R_D = (3.64 \pm 0.17) \times 10^{-3}$, $x'^2 = (0.18^{+0.21}_{-0.23}) \times 10^{-3}$, and $y' = (0.6^{+4.0}_{-3.9}) \times 10^{-3}$. The shape of the CL contours in $(x'^2, y')$ space was more or less similar to that from their earlier publication [69] shown in Fig. 20, but more restricted.

First evidence for mixing was reported in early 2007 by BaBar [78] and Belle [79], in the same issue of Physical Review Letters, using two different approaches. Both used methods developed previously.
### Figure 21: The number of $D^0 \to K^+\pi^-$ (DCS) decays as a function of $\Delta m$. The data points and statistical uncertainty bars are taken from the $K\pi$ slice fits. The shaded regions are determined from a least-squares fit and show the contributions from signal (dark gray) and random tagging pion background (light gray) as explained in the text. (Figure 2 in [76])

**BABAR** updated their WS/RS $K\pi$ analyses from 57 fb$^{-1}$ of data to 384 fb$^{-1}$. As **Belle** had the previous year, they fit the data under three hypotheses: mixing with no CPV, mixing allowing full CPV, and no mixing. Using a maximum likelihood ($\mathcal{L}$) fit, they reported that the “value of $-2\Delta \ln \mathcal{L}$ for no mixing [with respect to the central value observed] is 23.9 units. Including the systematic uncertainties, this corresponds to a significance equivalent to 3.9 standard deviations.” Their CL contour plot is shown as Fig. ??.

From the fit for mixing with no CPV, they found $R_D = (3.03 \pm 0.16 \pm 0.10) \times 10^{-3}$, $x^2 = (-0.22 \pm 0.30 \pm 0.21) \times 10^{-3}$, and $y' = (9.7 \pm 4.4 \pm 3.1) \times 10^{-3}$. The precisions are very close to those reported the previous year by **Belle**, yet significantly exclude the no-mixing hypothesis. To help visualize these results, the ratio of WS to RS events was plotted as a function of the measured decay time, as shown Fig. ??.

The detector’s typical decay time resolution was about half a $D^0$ lifetime which explains the large number of events observed at negative decay times. The increasing ratio as a function of decay time is the signature for mixing with either a positive value for the interference term (proportional to $y'\Gamma t$) or a dominant pure mixing term (proportional to $r_M\Gamma t^2 = (x^2 + y^2)(\Gamma t^2)/2$).

**Belle** collaboration updated their lifetime difference analysis from 25 fb$^{-1}$ of data to 540 fb$^{-1}$ []. They measured the apparent lifetimes of $D^0$ to $K^-\pi^+$, $K^-K^+$, and $\pi^-\pi^+$ samples using events consistent with the decay chain $D^{*+} \to D^0\pi^+$. They reported the difference between the lifetime of the $CP$ eigenstate samples and the lifetime of the $K^-\pi^+$ sample to be $y_{CP} = (1.31 \pm 0.32 \pm 0.25)\%$, 3.2 standard deviations from zero. The results of their simultaneous fit to the three decay time distributions and the ratio of the of the decay time distributions between the $D^0 \to K^-K^+$, $\pi^-\pi^+$ and $D^0 \to K^-\pi^+$ decays is illustrated in Fig. 24. Separate fits to the $D^0$ and $\bar{D}^0$ samples found asymmetries between the decay rates of the two $CP$ eigenstates compatible with zero: $A_\Gamma = (0.15 \pm 0.35 \pm 0.15)\%$ and $A_\Gamma = (-0.28 \pm 0.52 \pm 0.15)\%$ for the $K^-K^+$ and $\pi^-\pi^+$ samples, respectively.

The **BABAR** and **Belle** results were presented publicly at the Rencontres de Moriond Electroweak conference in early March 2007, shortly after they were submitted for publication. The Heavy Flavor Averaging Group (HFAG) reported world average results for charm mixing and CP violation parameters at the Flavor Physics and CP Violation conference in May 2007 (FPCP2007) [80] using statistical techniques described more fully in Ref. [81]. As an example of their results, Fig. 25 shows the likelihood contours presented at FPCP2007 assuming no CP violation and that $y$ measured from $D^0 \to K_S^0\pi^-\pi^+$ corresponds to $[\Gamma(CP^+) - \Gamma(CP^-)]/(2\Gamma)$. The null hypothesis is excluded at the $5\sigma$
Figure 22: The central value (point) and confidence-level (CL) contours for $1 - \text{CL} = 0.317$ ($1\sigma$), $4.55 \times 10^{-2}$ ($2\sigma$), $2.70 \times 10^{-3}$ ($3\sigma$), $6.33 \times 10^{-5}$ ($4\sigma$) and $5.73 \times 10^{-7}$ ($5\sigma$), calculated from the change in the value of $-2 \ln \mathcal{L}$ compared with its value at the minimum. Systematic uncertainties are included. The no-mixing point is shown as a plus sign (+). NB: this arXiv fig does not look the same as paper fig (Figure 3 in [78]).

Figure 23: The WS branching fractions from independent $\{m_{K\pi}, \Delta m\}$ fits to slices in measured proper time (points). The dashed line shows the expected wrong-sign rate as determined from the mixing fit shown in Fig. ???. The $\chi^2$ with respect to expectation from the mixing fit is 1.5; for the no-mixing hypothesis (a constant WS rate), the $\chi^2$ is 24.0. (Figure 4 in [78]).

The HFAG summary presented at FPCP2007 incorporated two results which had been shown publicly but which were published subsequently. $\text{BaBar}$ had searched for mixing using $D^{*+} \to D^0 \pi^+$ decays with $D^0 \to K^{(*)} \ell^+ \nu$ where the flavor of the neutral $D$ at production was tagged by both the charge of the $\pi$s and also by the flavor of a fully reconstructed second charm decay in the same event [82]. From 384 fb$^{-1}$ of data, 3 WS candidates were observed where 2.85 background events were expected. The central value of the mixing rate was reported to be $r_{\text{mix}} = 0.4 \times 10^{-4}$ with frequentist 68% and 90% confidence levels ($-5.6, 7.4) \times 10^{-4}$ and ($-13, 12) \times 10^{-4}$. The double-tag sample included about 1% of the RS signal events that would have been included in a more traditional (single-tag) semileptonic search for mixing [72, 70, 73], and produced about the same sensitivity per unit luminosity. Conceptually, both approaches could be used with the same data set. In this case, keeping the single-tag and double-tag samples disjoint would effectively double the statistics.

Belle’s time-dependent amplitude analysis of $D^0 \to K_{S}^0 \pi^- \pi^+$ from 540 fb$^{-1}$ of data [83] provided a more interesting result: assuming negligible CP violation, they measured the mixing parameters to
Figure 24: Results of the simultaneous fit to decay time distributions of (a) $D^0 \rightarrow K^+K^-$, (b) $D^0 \rightarrow K^-\pi^+$ and (c) $D^0 \rightarrow \pi^+\pi^-$ decays. The cross-hatched area represents background contributions, the shape of which was fitted using $M$ sideband events. (d) Ratio of decay time distributions between $D^0 \rightarrow K^+K^-,\pi^+\pi^-$ and $D^0 \rightarrow K^-\pi^+$ decays. The solid line is a fit to the data points. (Figure 2 in [79])

be $x = (0.80 \pm 0.29_{-0.07}^{+0.09} \pm 0.10,\%$ and $y = (0.33 \pm 0.24_{-0.06}^{+0.08,}\%$, where the errors are statistical, experimental systematic, and systematic due to the amplitude model used, respectively. The 95% confidence level contour, almost elliptical, just excluded the null hypothesis. Although this result did not provide as strong evidence for mixing as did the earlier BaBar [78] and Belle [79] results, it provided the first direct measurement of $(x,y)$ with fraction of a percent precision in each parameter. This measurement of $x$ anchored the world average value for several years, and the central value of $y$ was below that reported for $y_{CP}$. These observations were especially interesting in light of the only quantitative Standard Model predictions for mixing parameters in the $O(1\%)$ range which preceded the 2007 experimental evidence [84, 85]. The latter paper had specifically suggested that “if $y$ is in the ballpark of $+1\%$ as expected if the 4 $P$ states dominate $y$ [84], then we should expect $|x|$ between $10^{-3}$ and $10^{-2}$, and that $x$ and $y$ are of opposite sign.”

Figure 25: HFAG world average likelihood contours assuming no CP violation, presented at FPCP2007 [80].
In early 2008 CDF published an analysis of the time-dependence of the WS/RS $D^0 \to K\pi$ signal rates observed in 1.5 fb$^{-1}$ of data \cite{86}. The events were selected as in their 2006 time-integrated WS/RS ratio analysis \cite{76} (which had used only 0.35 1.5 fb$^{-1}$ of data). The data was divided into 20 bins of $t/\tau$ with bin size increasing from 0.25 to 2.0 to reduce the statistical uncertainty at higher decay times. In each decay time bin the data was divided into $\Delta m$ bins. In each $\Delta m$ bin a maximum likelihood fit to the $K\pi$ mass distribution was used to extract the $D^0$ signal. Non-prompt charm from $B \to D^* \to D^0$ was identified using the impact parameter of the $D^0$ flight direction with respect to the primary vertex, and subtracted statistically to avoid problems related to measuring decay time from the $B$ vertex. Sources of systematic uncertainty were parameterized, and these parameters allowed to float, along with the the signal and background levels, so that the reported fit errors accounted for systematic errors in addition to statistical errors. The observed WS/RS ratio approximately doubled over the range $0 < t/\tau < 10$. The central values from the fit were $R_D = (3.04 \pm 0.55) \times 10^{-3}$, $x'^2 = (-0.12 \pm 0.35) \times 10^{-3}$, and $y' = (8.5 \pm 7.6) \times 10^{-3}$. The error ellipse was remarkably similar to that shown the year before by BaBar, and the likelihood fit rejected the no-mixing hypothesis at the level of 3.8 standard deviations.

BaBar published an updated lifetime difference analysis, later in the year \cite{87}, using 384 fb$^{-1}$ of data. To expedite the publication, they updated only the tagged $D$ results. From this sample they reported ($y_{CP} = 1.24 \pm 0.39 \pm 0.13$)$\%$, confirming the Belle results \cite{79}. Combining their new tagged result with their previous measurement of the separate (untagged) sample of $D^0 \to K\pi\pi$ events, BaBar obtained $y_{CP} = (1.03 \pm 0.33 \pm 0.19)$%.

As the asymmetric $B$-factories Belle and BaBar had surpassed the integrated luminosity of CLEO at the $\Upsilon(4S)$ several years after turning on, CESR was reconfigured to operate near the charm threshold and the CLEO-c collaboration formed to exploit the new opportunities. Data taken at the $\Psi(3770)$ was used to study quantum-correlated final states to extract the relative strong phase between CF and DCS $D^0 \to K\pi$ amplitudes which relates ($x',y'$) to ($x,y$) \cite{88}. Because CESR was a symmetric machine, CLEO-c was sensitive only to the time-integrated terms in the correlated decay rates described by Eqn. 16. The effective branching fractions measured in correlated decays (called DT modes in \cite{88}) differ from those measured in flavor-tagged decays (called ST modes), especially when one of the decay products is a CP eigenstate, as can be seen by considering the amplitudes described by Eqs. 14. The relationships between effective branching fractions measured in correlated and uncorrelated decays for a variety of channels is shown in Table 3.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Correlated</th>
<th>Uncorr.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K^+\pi^-$</td>
<td>$1 + R_{WS}$</td>
<td>$1 + R_{WS}$</td>
</tr>
<tr>
<td>$S_\pm$</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>$K^+\pi^-, K^-\pi^+$</td>
<td>$R_M$</td>
<td>$R_{WS}$</td>
</tr>
<tr>
<td>$K^+\pi^+, K^+\pi^-$</td>
<td>$(1 + R_{WS})^2 - 4r \cos \delta (r \cos \delta + y)$</td>
<td>$1 + R_{WS}^2$</td>
</tr>
<tr>
<td>$K^+\pi^-, S_\pm$</td>
<td>$1 + R_{WS} \pm 2r \cos \delta \pm y$</td>
<td>$1 + R_{WS}$</td>
</tr>
<tr>
<td>$K^+\pi^-, e^-$</td>
<td>$1 - ry \cos \delta - rx \sin \delta$</td>
<td>1</td>
</tr>
<tr>
<td>$S_\pm, S_\pm$</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>$S_\pm, S_-$</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>$S_\pm, e^-$</td>
<td>$1 \pm y$</td>
<td>1</td>
</tr>
</tbody>
</table>

Using many tens of thousands of ST decays and several thousand DT decays from 281 pb$^{-1}$ of data, CLEO-c made a first determination of the strong phase $\delta$: $\cos \delta = 1.03_{-0.31}^{+0.34} \pm 0.06$. Incorporating
mixing parameter measurements from other experiments, they reported \( \cos(\delta) = 1.10 \pm 0.35 \pm 0.07 \) as well as \( x \sin \delta = (4.4^{+2.7}_{-1.8} \pm 2.9) \times 10^{-3} \) and \( \delta = (22^{+11+9}_{-12-11})^\circ \).

\( \text{BaBar} \) reported a measurement of mixing parameters from a study of \( D^0 \rightarrow K^+\pi^-\pi^0 \) [89] in 2009. Event selection was very similar to the 2006 analysis [75], but the sample increased from 230 fb\(^{-1}\) to 384 fb\(^{-1}\) and a full time-dependent amplitude analysis was performed. Conceptually, this was similar to the time-dependent analyses of \( D^0 \rightarrow K^0_S\pi^+\pi^0 \) pioneered by CLEO [74] and later used by Belle to measure \( x \) and \( y \) directly [83], although with two important, and related, variations on the theme. Because this channel has a single charged kaon, there are RS and WS samples corresponding to amplitudes which are predominantly CF and DCS, respectively. In addition, there are no quasi-two-body amplitudes with well defined CP symmetry. Thus, there is an overall strong phase difference between the DCS and mixed amplitudes which cannot be determined from this channel alone. It is analogous to the strong phase difference which appears in \( D^0 \rightarrow K^+\pi^- \), but the value of the phase here, \( \delta_{K\pi\pi0} \), should be independent. The paper reported \( x'_{K\pi\pi0} = (2.61^{+0.57}_{-0.68} \pm 0.30)\% \) and \( y'_{K\pi\pi0} = (-0.06^{+0.55}_{-0.64} \pm 0.34)\% \), a result inconsistent with the no-mixing hypothesis with a significance of 3.2 standard deviations. These central values correspond to \( r_{mix} \approx 3.4 \times 10^{-4} \), a higher mixing rate than inferred from other measurements, but still below the limit established from semileptonic mixing searches.

Belle reported improved limits on mixing from a study of \( D^0 K\mu\nu \) and \( D^0 \rightarrow K\nu \) samples using 492 fb\(^{-1}\) of data [90]. This was the first search for mixing using \( D^0 K\mu\nu \) decays at an e\(^+\)e\(^-\) machine. The statistics were about 15\% lower than for \( D^0 \rightarrow K\nu \). The \( K\mu\nu \) and the \( K\nu \) data were analyzed separately, and these data sets were subdivided according to two running periods with significantly different detector configurations. The results from these four disjoint data sets were then combined to produce a mixing rate \( r_{mix} = (1.3 \pm 2.2 \pm 2.0) \times 10^{-4} \) and a 90\% CL limit \( r_{mix} < 6.1 \times 10^{-4} \).

Belle also reported a measurement of \( y_{CP} \) from an untagged sample of \( D^0 \rightarrow K^0_S K^- K^+ \) using 673 fb\(^{-1}\) in 2009 [91]. Rather than doing a full time-dependent amplitude analysis, they took advantage of the very clean \( CP \)-odd signal in the \( \phi K^0_S \) band of the Dalitz plot and compared the average decay time of signal events there with the average decay time of signal events in regions of the Dalitz plot associated with three flavor-specific intermediate states: \( K^- a_0(980)^+ \), \( K^- a_0(1450)^+ \), and \( K^+ a_0(980)^- \). The analysis used the time-integrated amplitude model published by \( \text{BaBar} \) the previous year [92] as part of a measurement of the CKM angle \( \gamma \) [92] to define the \( \phi K^0_S \) and flavor-specific regions. With \( \approx 72 \times 10^3 \phi K^0_S \) events and \( \approx 62 \times 10^3 \) flavor-specific events, they obtained \( y_{CP} = (+0.11 \pm 0.61 \pm 0.52)\% \). The dominant sources of systematic uncertainty related to the decay time resolution functions of the \( \phi K^0_S \) and flavor-specific samples and variations in the selection criteria.

In 2009 \( \text{BaBar} \) updated its lifetime difference analysis again, this time extending its study of the untagged \( K^- K^- \) and \( K^- \pi^+ \) samples to use 384 fb\(^{-1}\) of data [93]. The signal yields in these channels were \( \approx 254 K \) and \( \approx 2710 K \), respectively, compared to the \( \approx 70 K \) and \( \approx 731 K \) found in the tagged analysis using the same luminosity [87], providing significantly better statistical precision. However, the signal purities were lower, \( \approx 81\% \) and \( \approx 94\% \), compared to \( \approx 99.6\% \) and \( \approx 99.9\% \). Consequently, the systematic uncertainty from the untagged sample was greater than that from the tagged sample, although still somewhat small than the statistical error. The result from the untagged sample only was \( y_{CP} = (1.12 \pm 0.26 \pm 0.22)\% \). The dominant sources of systematic uncertainty related to the probability density functions used to describe background events in the signal region. The events in the untagged and tagged samples were independent by construction, but the systematic uncertainties judged to be 100\% correlated. Combining the results of the untagged and tagged analyses, the paper reported \( y_{CP} = (1.16 \pm 0.22 \pm 0.18)\% \). Summing the statistical and systematic uncertainties in quadrature, the no mixing hypothesis was excluded with 4.1 \( \sigma \) significance.

In 2010 \( \text{BaBar} \) reported a measurement of mixing parameters from time-dependent amplitude analyses of \( D^0 \rightarrow K^0_S \pi^- \pi^+ \) and \( D^0 \rightarrow K^0_S K^- K^+ \) decays using \( \approx 470 \mathrm{fb}^{-1} \) of data [94]. The amplitudes were described as coherent sums of quasi-two-body amplitudes conceptually similar to those used
earlier by CLEO [74] and Belle [83]. However, rather than using only Breit-Wigner (BW) propagators, the $\pi\pi$ S-wave dynamics was described using a $K$-matrix formalism and the $K\pi$ S-wave dynamics was described using a BW for the $K_0^*(1430)^\pm$ with a coherent nonresonant contribution parameterized by a scattering length and effective range similar to those used to described $K\pi$ scattering data from the LASS experiment [95]. For the $K\bar{K}$ S-wave, a coupled BW was used for the $a_0(980)$ isovector with BWs for the $f_0(1370)$ and $a_0(1450)$ states. The nominal mixing fit results, in which the $D^0$ and $\bar{D}^0$ samples from $D^0 \to K^0_S\pi^-\pi^+$ and $D^0 \to K^0_SK^-K^+$ were combined are shown in Table 1, along with separate results for the $D^0$ and $\bar{D}^0$ samples and for the $D^0 \to K^0_S\pi^-\pi^+$ and $D^0 \to K^0_SK^-K^+$ samples. In each case, statistical errors, systematic uncertainties related to the data, and systematic uncertainties related to the amplitude models are reported. The precision of the $K^0_S\pi^-\pi^+$ analysis was very similar to that of the earlier Belle analysis [83], but the 68.3% CL error ellipses from the two experiments did not overlap and the central values from each experiment barely touched the 95% CL contours from the other experiment.

Table 4: Results from the mixing fits. The first uncertainty is statistical, the second systematic, and the third systematic from the amplitude model. For the nominal fit, the corresponding correlation coefficients between $x$ and $y$ are 3.5%, 16.0% and −2.7%, respectively.

<table>
<thead>
<tr>
<th>Fit type</th>
<th>$x/10^{-3}$</th>
<th>$y/10^{-3}$</th>
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</thead>
<tbody>
<tr>
<td>Nominal</td>
<td>1.6 ± 2.3 ± 1.2 ± 0.8</td>
<td>5.7 ± 2.0 ± 1.3 ± 0.7</td>
</tr>
<tr>
<td>$K^0_S\pi^-\pi^+$</td>
<td>2.6 ± 2.4</td>
<td>6.0 ± 2.1</td>
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<tr>
<td>$K^0_SK^-K^+$</td>
<td>−13.6 ± 9.2</td>
<td>4.4 ± 5.7</td>
</tr>
<tr>
<td>$D^0$</td>
<td>0.0 ± 3.3</td>
<td>5.5 ± 2.8</td>
</tr>
<tr>
<td>$\bar{D}^0$</td>
<td>3.3 ± 3.3</td>
<td>5.9 ± 2.8</td>
</tr>
</tbody>
</table>
References


