Edge Magic Labeling for Unions of Cycles

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Definition

An *Edge Magic Total (EMT) labeling* is a bijection from the set of vertices and edges to a set of numbers defined by $\lambda : V \cup E \rightarrow \{1, 2, ..., |V| + |E|\}$ with the property that for every $xy \in E$, $\lambda(x) + \lambda(y) + \lambda(xy) = k$ for some integer k.

A *Super Edge Magic Total (SEMT) labeling* is EMT labeling with $\lambda(V) = \{1, 2, ..., |V|\}$ and $\lambda(E) = \{|V| + 1, |V| + 2, ..., |V| + |E|\}$.

Known Results

Cycles C_n are known to have EMT labeling for $n \geq 3$ and a SEMT labeling iff $n \geq 3$ is odd. One among a few variations of cycles that is known to have EMT labeling is the union of cycles.

Figuera-Centeno *et al.* proved that the union of m identical cycles (mC_n) has a SEMT labeling if and only if both m and n are odd. No results had been found for other parities of m and n.

Our objective is to investigate the EMT labeling for mC_n when m and n are not both odd.

In what follows, mC_n where $C_i = (V_i, E_i)$, and let $V_i = \{v_i^1, v_i^2, ..., v_i^n\}$ and $E_i = \{e_i^1, e_i^2, ..., e_i^n\}$ with e_i^j denotes the edge joining the vertices v_i^j and v_i^{j+1} where the index j is taken modulo n.

References

[1] Gallian, J.A., A Dynamic Survey of Graph Labeling, The Electric Journal of Combinatorics, 2013.

[2] Sugeng, K.A., Magic and Antimagic Labeling of Graphs, Dissertation, University of Ballarat, 2005.

[3] Figuera-Centeno, R.M., Ichisima, R., Muntaner-Batle, On Super Edge-Magic Graphs, Ars Combinatoria.64, 2002.

[4] Figuera-Centeno, R.M., Ichisima, R., Muntaner-Batle, F.A., Oshima, A., A Magical Approach to Some Labeling Conjectures, Discussiones Mathematicae.31, 2011.

[5] Wijaya, K., Baskoro, E.T., Edge-Magic Total Labeling on Disconnected Graphs, Proceedings of the eleventh AWOCA, 2000.

[6] Marr, A.M., Wallis, W.D. Magic Graphs, Springer, 2013.

Kotzig Array

A Kotzig Array is a $d \times m$ grid, each row being a permutation of $\{0, 1, , m-1\}$ and each column having a constant column sum.

Algorithm

For mC_n with odd m and even n, the EMT labeling construction can be obtained using the following steps:

1. Make m copies of labeled C_n as given in [6]. Reassign the vertex labeled 1 in [6] as v_1^1 and assign the rest of vertices be v_i^j with j ordered clockwise.
2. Make $3 \times m$ Kotzig array, multiply each number inside the array with $2n$.
3. Add the number on the i^{th} column of the first row to the label of odd ordered vertices (vertices with odd j) in the i^{th} cycle of mC_n.
4. Add the number on the i^{th} column of the second row to the label of even ordered vertices (vertices with even j) in the i^{th} cycle of mC_n.
5. Add the number on the i^{th} column of the third row to the label of the edges in the i^{th} cycle.

For convenience in describing the example later, the algorithm above is only for times when we start using a single cycle. The generalization of this algortihm can be made using small modification on the first two steps.

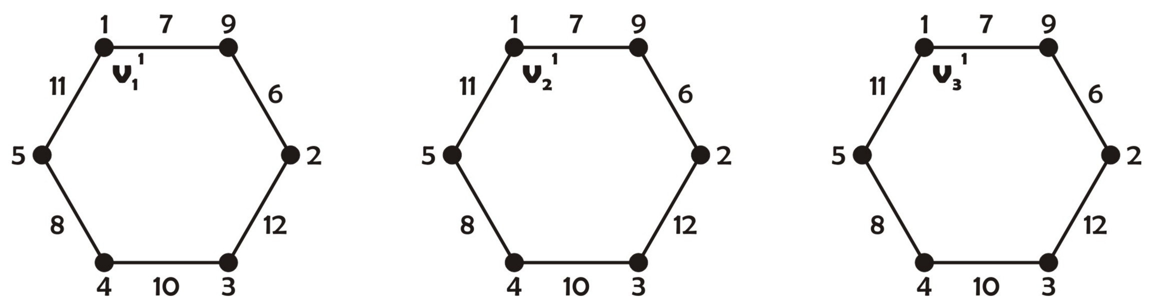
Observe that every integer m can be expressed as a multiplication between a power of two and an odd number.

For the first step, make $\frac{m}{2}$ copies of labeled $mC_n, m = 2^a$ for $a \in \mathbb{N}$. For the second step, make $3 \times \frac{m}{a+1}$ Kotzig array, multiply each number inside the array with $2^{a+1}n$.

Examples

Below is how the algorithm applied to construct the labeling of $3C_6$:

Step 1.



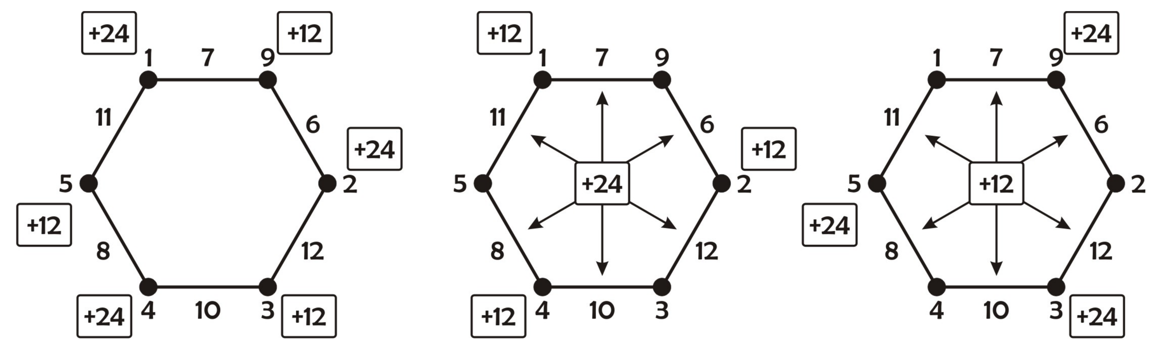
Step 2.

2	1	0
1	0	2
0	2	1

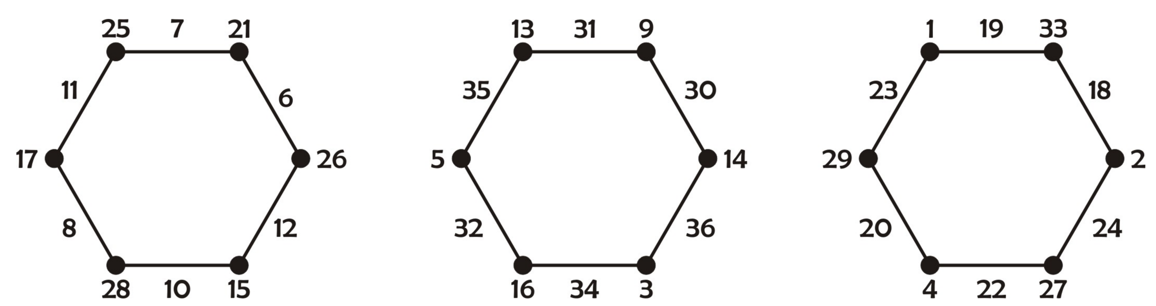
x 12

24	12	0
12	0	24
0	24	12

Step 3 to 5.

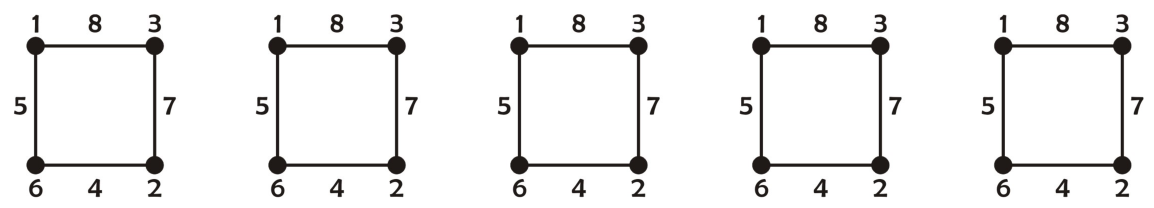


EMT labeling for $3C_6$.



Below is how the algorithm applied to construct the labeling of $5C_4$:

Step 1.



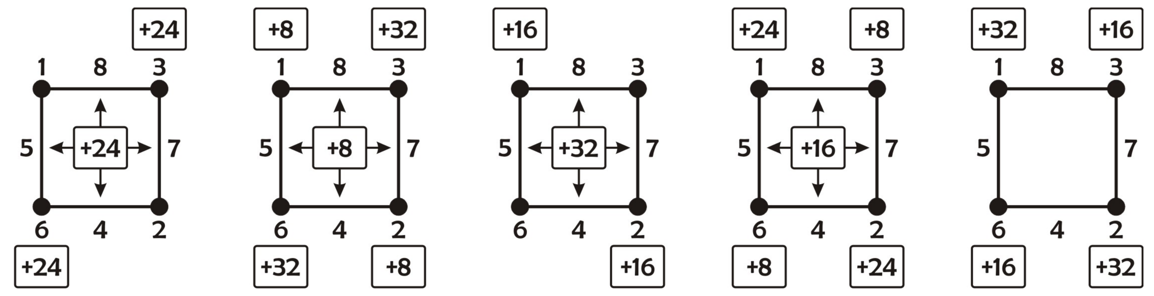
Step 2.

0	1	2	3	4
3	4	0	1	2
3	1	4	2	0

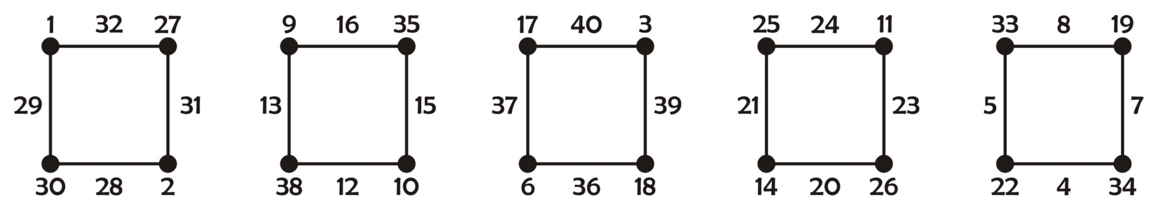
x 8

0	8	16	24	32
24	32	0	8	16
24	8	32	16	0

Step 3 to 5.



EMT labeling for $5C_4$.



Conclusion

m	n	Type of labeling	Notes
Odd	Odd	SEMT	all odd n and m (result from [3])
Odd	Even	EMT	all $m \geq 1$ and n 2
Even	Even	EMT	$m \equiv 2 \bmod 4, n \in \{4, 6, 8, 10, 12\}$ and $m \equiv 4 \bmod 8, n = 4$
Even	Odd	not EMT	if $m = 2, n = 3$. others are unknown

Table 1: Known EMT/SEMT labeling for unions of cycles

Future Research

We strongly believe that $2C_n$ has EMT labeling with $k = 5n + 2$ for every even value of n. Currently we are still trying to generate EMT labeling for $mC_n, m = 2^a, a \in \mathbb{N} \cup \{0\}$ in oder to prove that mC_n has EMT labeling for every value of m and n.

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