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Research Statement

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Overview

My research focuses on graph theory problems that are motivated by real-life applications. My interest in this topic was piqued by the first mathematical project I worked on. This project was about DNA sequencing, where we used DNA graph labeling to reconstruct readable DNA strands from smaller pieces obtained from an experiment. I work on various problems related to graph labelings, which include magic and antimagic (edge- and/or vertex-)labelings of graphs in addition to DNA graph labelings. I also work on problems related to the diameter of graphs, which are motivated by problems arising in social and transportation networks. The goal of these projects is to give a general upper bound on the diameter of a graph in terms of other parameters such as the number of vertices, minimum degree and chromatic number of the graph.

Both distance problems and labeling problems in graph theory are abound with potential projects suitable for the undergraduate audience. Graph theory itself can be viewed as a study of relations between certain objects and the study of network structures. Studying important properties of networks and applying the results to real-world networks like Facebook, the internet, etc. are problems that can easily motivate undergraduates. I already have a "problem garden" related to my research projects that I look forward to collaborating on with interested students.

Diameter of 3-colorable Graph

The distance between two vertices in a graph is the number of edges in a shortest path connecting them. The diameter of a graph G is the greatest distance between any pair of vertices of G. Problems related to distances in graphs are often originate from distances in networks. One recent motivation from networks in social media is given in Chakraborty et al. [3].

Let us consider a simple, finite, connected graph $G = (V, E)$ on n vertices, with minimum degree $\delta > 1$, and diameter $\operatorname{diam}(G)$. The natural problem of bounding $\operatorname{diam}(G)$ in terms of n and δ was independently solved by several authors [1, 10, 12, 19] who stated that for a fixed $\delta \geq 2$ and large n, $\operatorname{diam}(G) \leq \frac{3n}{\delta+1} + O(1)$. In 1989, Erdős, Pach, Pollack, and Tuza [10] conjectured that this upper bound on the diameter can be improved if G does not contain a complete subgraph K_n when n is large:

(i) If G is K_{2r}-free and δ is a multiple of $(r-1)(3r+2)$ then $\operatorname{diam}(G) \leq \frac{2(r-1)(3r+2)}{(2r^2-1)\delta} n + O(1)$.

(ii) If G is K_{2r+1}-free and δ is a multiple of $3r-1$ then $\operatorname{diam}(G) \leq \frac{3r-1}{r\delta} n + O(1)$.

They constructed examples that show that this conjecture, if true, is the best possible.

No progress on the above conjecture on any values of r was reported until 2009 when Czabarka et al. [5] considered the somewhat weaker conjecture where the condition K_k-free is replaced with $(k-1)$-colorable, and showed that for every connected 4-colorable graph G of order n and minimum degree $\delta \geq 1$ we have $\operatorname{diam}(G) \leq \frac{5n}{2\delta} - 1$. We examined this colorability version of for different values of r, and found a counterexample for 3-colorable graphs, which is naturally also a counterexample for the original conjecture on K_4-free graphs.

We constructed a family of 3-colorable graphs on n vertices with minimum degree δ and diameter $\frac{7n}{3\delta}$. Since for $\delta > 48$ this is larger than the upper bound conjectured by Erdös et al., this effectively shows that the conjecture is not true for K_4-free graphs. We conjecture ([6]) that for every connected 3-colorable graph G

of order n and minimum degree $\delta \geq 1$, we have $\text{diam}(G) \leq \frac{7n}{3\delta} + O(1)$. Our approach to prove our conjecture used Linear Programming and various versions of the Inclusion-Exclusion principle. We already have a result that gives an upper bound on the diameter that is smaller than for the 4-colorable case, but there is still a gap between the conjectured and proven upper bounds. This method also yields a straightforward proof for the earlier result on 4-colorable graphs without doing the case by case construction analysis as in by Czabarka et al. [5].

Related Work

É. Czabarka, I. Singgih, L.A. Székely, Diameter of 3-colorable graph, *in preparation*.

Ongoing Project

I will extend our arguments on the modified Erdős et al. conjecture for larger values of r. This research has two directions: proving the conjectured upper bounds (or potentially some modified conjectured values) and trying to come up with constructions that may disprove some of the upper bounds, just as we have done in the 3-colorable case.

Midrange Crossing Constant

Let $\kappa(n, e)$ be the minimum crossing number of graphs with n vertices and at least e edges. The crossing lemma implies that for $e > 4n$, the value $\kappa(n, e)\frac{n^2}{e^3}$ is bounded below by a positive constant. Erdős and Guy [9] conjectured that for $n \ll e \ll n^2$, the limit of $\kappa(n, e)\frac{n^2}{e^3}$ exists as n goes to infinity. This conjecture was verified by Pach, Spencer and Tóth [20] where they called the limit C as *midrange crossing constant*. The exact value of C is unknown, and the current best bound is $\frac{1}{29} \leq C \leq \frac{8}{9\pi^2}$. The upper bound was shown by Pach and Tóth [21] by construction on a slightly perturbed $\sqrt{n} \times \sqrt{n}$ grid, where a pair of points are joined by a straight line segment if their distance is not greater than d.

With Czabarka, Székely, and Wang [7], we gave a spherical construction by uniformly randomly selecting n vertices on the unit sphere. Two vertices are connected by the shorter arc of their great circle if their great circle distance bounded by d. The constant follows from the first moment method as d is approaching zero. Our construction give the same bound as Pach and Tóth, with appreciably easier computation. It combines existing ideas in a novel, insightful way that may yield better to various generalizations.

Related Work

E. Czabarka, I. Singgih, L.A. Szekely, Z. Wang, Some remarks on the midrange crossing constant, to appear on *Studia Scientiarum Mathematicarum Hungarica* (2019), *arXiv:1907.00368v1*.

Ongoing Project

One challenging project is to better the bound for the midrange constant. I am also interested in problems on biplanar crossing numbers, especially biplanar crossing numbers of complete bipartite graphs. The crossing number of a complete bipartite graph (Zarankiewicz conjecture) is a variant of the Brick Factory Problem that were proposed by Paul Turán during WWII in a forced work camp. The definition of the biplanar crossing number is motivated by VLSI (Very Large Scale Integration) designs. The structural problems of constructing the possible drawings for bounding the biplanar crossing number of small graphs (see [8]) is an appropriate research topic for interested undergraduate students.

Graph Labelings

Graph labeling is an assignment of labels to the edge set E and/or vertex set V of a graph $G = (V, E)$. It is an important technique used in graph theory, for which applications can be found in many areas ranging from DNA sequencing to GPS programming. Graph labelings can have certain weights associated with each edge and/or vertex. Research in graph labelings has been growing rapidly since they were first introduced by Sedláček in 1963 [23]. Two main problems in graph labeling are to prove or disprove the existence of a certain labeling for certain families of graphs, and to provide a construction when they do exists. Different variations of graph labelings have been defined, and hundreds of publications resulted on these variations. The papers [11] and [29] provide good surveys on the existing results. I work on several variations of graph labelings, and I follow the development on their new variations.

DNA Graphs

Sequencing by Hybridization is one of the methods-in-development of DNA sequencing in which its computational phase uses DNA graphs to reconstruct DNA strands. A digraph (or directed graph) $D = (V, A)$ is said to be (α, k)-labeled if it is possible to label each vertex x of D with a label $(l_1(x), l_2(x), \ldots, l_k(x))$ of length k such that $l_i(x) \in \{1, 2, \ldots, \alpha\}$ for all $x \in V$ and $i \in \{1, 2, \ldots, k\}$, each vertex has different labels, and the deBruijn property holds, i.e. $xy \in A \Leftrightarrow l_j(x) = l_{j-1}(y)$ for each $j \in \{2, 3, \ldots, k\}$. A DNA graph is a digraph D which is (α, k)-labeled for some integers $\alpha \leq 4$ and $k > 1$. Figure 1 shows the example of a digraph with a $(2, 3)$-labeling.

The ultimate goal is to find a Hamiltonian path in our (α, k)-labeled graph because this allows us to merge the overlapping labels into a single strand. The motivation of this approach is that in many DNA sequencing applications, shorter segments of DNA are created that then need to be merged into (the original) long sequence. The graph in Figure 1 has such a Hamiltonian path, and Figure 2 shows that path with the vertices now labeled by translating label "1" into C (cytosine) and label "2" to T (thymine) in Figure 1. This Hamiltonian path describes the ordering of the spectrum TCC, CCT, CTC, TCT, CTT which results in the longer DNA strand TCCTCTT by merging the overlapping nucleotides.

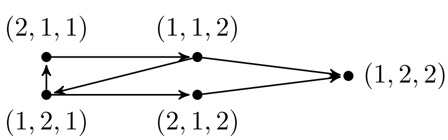

Figure 1: Digraph with a $(2, 3)$-labeling.

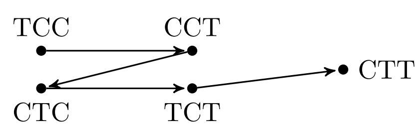

Figure 2: DNA spectrum derived from Figure 1.

Finding a family of DNA graph is NP hard [22]. Li and Zhang [17] were the first to translate this problem of DNA sequencing into mathematical terms. They found that directed paths, cycles, rooted trees, and self adjoint digraphs are DNA graphs, and they came up with the idea that line digraphs of certain DNA graphs might be also be DNA graphs. With K.A. Sugeng, we continued this work on directed cycles with one chord. Using line digraphs, we extended the results into infinite families of graphs [25]. I continue this research for more families of graphs, the results and the open problems that emerged from them can be found in [27].

Related Work

I. Singgih, K.A. Sugeng, D.R. Silaban, DNA Graph Characterization for Line Digraph of Dicycle with One Chord, *AKCE International Journal of Graphs and Combinatorics*, **10:2**(2013), pp. 157–167.

I. Singgih, Edge Magic Total Labeling of Lexicographic Product $C_{4(2r+1)} \circ \overline{K_2}$, Cycles with Chords, Unions of Paths, and Unions of Cycles and Paths, *Indonesian Journal of Combinatorics*, **2:2**(2018), pp. 111–122.

I. Singgih, DNA graph characterization for the line digraph of dicycle with $\lfloor \frac{n}{3} \rfloor$ chords, ∞-digraph $C_n \cdot C_p$, and 3-blade propeller $C_n \cdot C_p \cdot C_q$, *arXiv:1812.02880.*

Ongoing Project

Finding others families of DNA graphs is an open problem suitable for undergraduate research. I am also interested for interdisciplinary collaborations with students and/or colleagues who have interest in bioinformatics to explore possible applicable labelings to other DNA sequencing methods.

Edge/Vertex Magic Total Labelings

An *edge magic total* (EMT) labeling of a graph $G = (V, E)$ is a bijection $\lambda : V \cup E \to \{1, 2, \ldots, |V|+|E|\}$ such that for every $xy \in E$ we have the edge weight $w(xy) = \lambda(xy) + \lambda(y) + \lambda(x)$ equals to an integer constant. A *vertex magic total* (VMT) labeling of a graph $G = (V, E)$ is a bijection $\lambda : V \cup E \to \{1, 2, \ldots, |V| + |E|\}$ such that for every $x \in V$ the edge weight $w(x) = \lambda(x) + \sum_{xy \in E} \lambda(xy)$ equals to an integer constant. If the label of any vertex is smaller than the label of any edge, we call the EMT (or VMT) labeling to be *super* and denote it by SEMT or SVMT). Holden, McQuillan, and McQuillan [15] posed the strong conjecture that a 2-regular graph of odd order possesses a SVMT labeling iff it is not one of $C_4 \cup C_3, C_4 \cup 3C_3$, or $C_5 \cup 2C_3$.

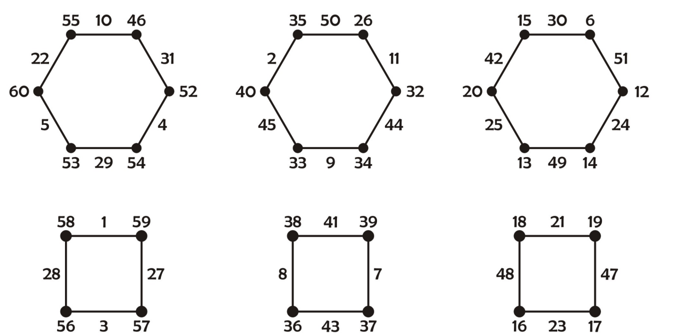

Figure 3: VMT labeling for $3C_6 \cup 3C_4$ with magic constant 87.

With D. Froncek and S. Cichaz [4], we came up with a method to extend the construction of VMT/SVMT for 2-regular graphs. Using our method we get VMT/SVMT of union of cycles mC_{nr} by extending VMT/SVMT of mC_n where r is an odd natural numbers. This is a significant step towards resolving the conjecture. We also answered several open problems for case when n is even. We later realized that the EMT/SEMT version of this method is also applicable to many other graphs that are not 2-regular [26].

Related Work

S. Cichacz, D. Froncek, I. Singgih, Vertex Magic Total Labeling for 2–regular Graphs, *Discrete Mathematics*, **340:1**(2017), pp. 3117–3124.

I. Singgih, Edge Magic Total Labeling of Lexicographic Product $C_{4(2r+1)} \circ \overline{K_2}$, Cycles with Chords, Unions of Paths, and Unions of Cycles and Paths, *Indonesian Journal of Combinatorics*, **2:2**(2018), pp. 111–122.

Ongoing Project

I will work towards resolving Holden, McQuillan, and McQuillan conjecture on 2-regular graphs. I will also seek other families of graphs in which our method can be applied to.

Subtractive Labeling

In his Master thesis [2], Barone extended magic labelings to directed graphs. While he stated some definitions and observations, no results of his thesis were published. I continued this work and also applied it to antimagic labeling. These labelings are motivated by the network flow problems.

A *subtractive arc-magic labeling* (SAML) of a directed graph $G = (V, A)$ is a bijection $\lambda : V \cup A \to \{1, 2, \ldots, |V| + |A|\}$ with the property that for every $xy \in A$ we have the subtractive arc weight $wt^-(xy) = \lambda(xy) + \lambda(y) - \lambda(x)$ equals to an integer constant μ. If arc weights are distinct for every $xy \in A$, then λ is a *subtractive arc-antimagic labeling* (SAAL). The subtractive labeling λ of G is called *strong* if $\lambda(V) = \{1, 2, \ldots, |V|\}$, and called *strong** if $\lambda(A) = \{1, 2, \ldots, |A|\}$.

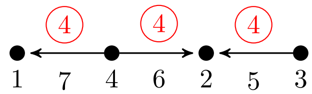

Figure 4: A strong SAML of P_4 with $\mu = 4$.

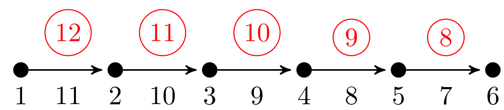

Figure 5: A strong SA$(8,1)$AL of P_6.

A *subtractive vertex-magic labeling* (SVML) of G is such bijection with the property that for every $x \in V$ we have the subtractive vertex weight $wt^-(x) = \lambda(x) + \sum \lambda(yx) - \sum \lambda(xy)$ equals to an integer constant for all $y \in V, xy \in A$. If vertex weights are distinct for every $x \in V$ and $y \in V, xy \in A$, then λ is a *subtractive vertex-antimagic labeling* (SVAL). If the distinct weights $wt^-(xy)$ or $wt^-(x)$ forms a sequence $a, a + d, a + 2d, \ldots$, then the labeling is denoted by SA(a, d)AL or SV(a, d)AL, respectively.

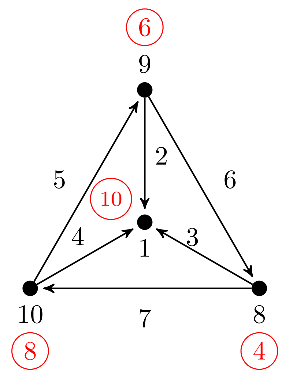

Figure 6: An SV$(4, 2)$AL of W_3.

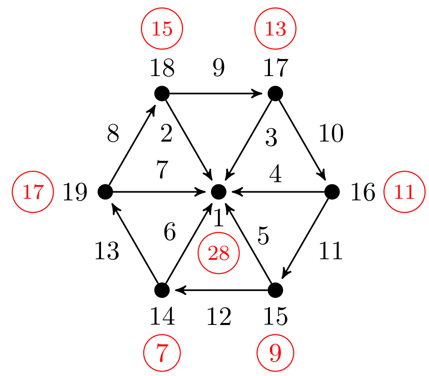

Figure 7: An SVAL of W_6.

I proved some results on the existence/non-existence of such labelings, and when they exist I constructed subtractive magic and/or subtractive anti-magic labelings for dipaths, dicycles, distars, directed wheels, directed (n, t)-tadpoles, directed friendship graphs, and ∞-digraphs [28]. The continuation of this work for other families of graphs is an ongoing project with P. John [16].

Related Work

I. Singgih, Subtractive Magic and Antimagic Total Labelings, *arXiv:1811.03033*.

P. John, I. Singgih, Subtractive Magic and Antimagic Total Labelings on Fans and Umbrellas, *in preparation*.

Ongoing Project

Due its novelty, the problem of finding subtractive labelings of other families of graphs is still wide open. Until now, there are no known families of graphs proven to have SVML. The broader goal is to have characterizations for graphs to admit subtractive labelings.

Antimagic Orientations for Directed Graphs

An *antimagic labeling* of a graph G is a bijection $\tau : E \to \{1, 2, \ldots, |E|\}$ such that distinct vertices has different weights, where the weight $w(x)$ of a vertex x is the sum of labels on edges incident to x. Some of the literature calls this labeling *vertex antimagic edge labeling*.

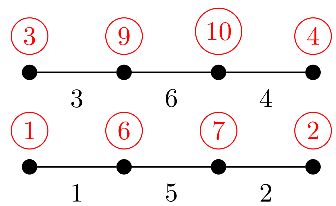

Figure 8: An antimagic labeling for $2P_4$.

Hartsfield and Ringel [13] introduced antimagic labeling in 1990 and conjectured that every connected graph other than K_2 is antimagic. Motivated by this Conjecture, Hefetz, Mütze, and Schwartz [14] introduced antimagic labeling of digraphs, where the weight $w'(x)$ of x is defined as the sum of labels on arcs going towards x minus the sum of labels on arcs going away from x. They also conjectured that every connected graph admits an antimagic orientation. This conjecture has been verified for regular graphs [14, 18] and biregular bipartite graphs with minimum degree at least two [24], but otherwise is wide open. With my collaborators in the Graduate Research Workshop in Combinatorics, we verified the conjecture for graphs with large maximum degree $\Delta = n - k$ for $k \leq 5$ [30].

Related Work

D. Yang, J.Carlson, A. Owens, K. E. Perry, I. Singgih, Z-X. Song, F. Zhang, X. Zhang, Antimagic orientations of graphs with large maximum degree, *arXiv:1908.06072*.

Ongoing Project

This project can develop to two directions. For characterization problem, we can try to extend the result in [30] for $\Delta = n - k$ with larger values of k. This invites collaboration with students having computational or programming backgrounds to help with numerous cases analysis. For constructional problems, I already have several families of graphs that are antimagic, such as union of paths mP_n and union of subdivided stars $mS_n(k)$. I will continue to generalize those results to other family of graphs.

Future Work

In addition on working on the stated ongoing projects, my immediate plan on graph labelings is to create a small survey and compile a list of remaining problems to work on. These problems can involve students with interests in pattern recognition and computer programming. I look forward to continuing the work with my current collaborators as well as beginning new projects with my colleagues and interested students.

References

[1] D. Amar, I. Fournier, A. Germa, Odre minimum d'un graphe simple de diametre, degré minimum et connexité donnés, *Ann Discrete Math.*, **17**(1983), pp. 7–10.

[2] C. A. Barone, Magic labelings of directed graphs, *Master thesis, University of Victoria* (2004).

[3] A. Chakraborty, Trina Dutta, Sushmita Mondal, Asoke Nath, Application of Graph Theory in Social Media. *International Journal of Computer Sciences and Engineering*, **6:10**(2018), pp. 722–729.

[4] S. Cichacz, D. Froncek, I. Singgih, Vertex Magic Total Labeling for 2–regular Graphs, *Discrete Mathematics*, **340:1**(2017), pp. 3117–3124.

[5] É. Czabarka, P. Dankelmann, L.A. Székely, Diameter of 4-colorable graphs, *European Journal of Combinatorics*. **30**(2009), pp. 1082–1089.

[6] É. Czabarka, I. Singgih, L.A. Székely, Diameter of 3-colorable graph, *in preparation.*

[7] É. Czabarka, I. Singgih, L.A. Székely, Z. Wang, Some remarks on the midrange crossing constant, accepted by *Studia Scientiarum Mathematicarum Hungarica* (2019), *arXiv:1907.00368v1.*

[8] É. Czabarka, O. Sýkora, L.A. Székely, I. Vrt'o, Biplanar Crossing Numbers I: A Survey of Results and Problems, *More Sets, Graphs and Numbers, Bolyai Society Mathematical Studies book series ISBN:978-3-540-32377-8*, **15**(2006), pp 55–77.

[9] P. Erdős, R.K. Guy, Crossing number problems, *Amer. Math. Monthly*, **80** (1973), pp. 52–58.

[10] P. Erdős, J. Pach, R. Pollack, Z. Tuza, Radius, Diameter, and Minimum Degree, *Journal of Combinatorial Theory*, **B47** (1989), pp. 73–79.

[11] J. Gallian, A Dynamic Survey of Graph Labeling, *The Electron. J. Combin.*, **21**(2018).

[12] D. Goldsmith, B. Manvel, V. Faber, A lower bound for the order of the graph in terms of the diameter and minimum degree, *J. Combin. Inform. Syst. Sci.*, **6**(1981), pp. 315–319.

[13] N. Hartsfield and G. Ringel, Pearls in Graph Theory, *Academic Press, Boston*, (1990), pp. 108–109 (revised version, 1994).

[14] D. Hefetz, T. Mutze, and J. Schwartz, On antimagic directed graphs, *J. Graph Theory* **64**(2010), pp. 219–232.

[15] Holden J., Mcquillan D., Mcquillan J.M, A conjecture on strong magic labelings of 2-regular graphs, *Discrete Math.*, **309**(2009), pp. 4130–4136.

[16] P. John, I. Singgih, Subtractive Magic and Antimagic Total Labelings on Fans and Umbrellas, *in preparation.*

[17] X. Li and H. Zhang, Characterization for Some Types of DNA Graphs, *J. Math.Chem.*, **42:2**(2006), pp 65–79.

[18] T. Li, Z-X. Song, G. Wang, D. Yang, C-Q. Zhang, Antimagic orientations of even regular graphs, *J. Graph Theory* **90**(2019), pp. 46–53.

[19] J.W. Moon, On the diameter of a graph, *Mich. Math. J.*, **12**(1965), pp. 349–351.

[20] J. Pach, J. Spencer, G. Tóth, New bounds on crossing numbers, *Discrete & Computational Geometry* **24:4**(2000), pp 623–644.

[21] J. Pach, G. Tóth, Graphs drawn with few crossings per edge, *Combinatorica* **17** (1997), pp427–439.

[22] R. Pendavingh, P. Schuurman, G.J. Woeginger, Recognizing DNA graphs is difficult, *Discrete Applied Mathematics*, **127:1**(2003), pp. 85–94.

[23] J. Sedlek, Problem 27, in: Theory of Graphs and its Applications, *Proc. Symposium Smolenice 1963, Prague* (1964), pp. 163–164.

[24] S. Shan, X. Yu, Antimagic orientation of biregular bipartite graphs, *Electron. J. Combin.*, **24**(2017): Paper 4.31.

[25] I. Singgih, K.A. Sugeng, D.R. Silaban, DNA Graph Characterization for Line Digraph of Dicycle with One Chord, *AKCE International Journal of Graphs and Combinatorics*, **10:2**(2013), pp. 157–167.

[26] I. Singgih, Edge Magic Total Labeling of Lexicographic Product $C_{4(2r+1)} \circ \overline{K_2}$, Cycles with Chords, Unions of Paths, and Unions of Cycles and Paths, *Indonesian Journal of Combinatorics*, **2:2**(2018), pp. 111–122.

[27] I. Singgih, DNA graph characterization for the line digraph of dicycle with $\lfloor \frac{n}{3} \rfloor$ chords, ∞-digraph $C_n \cdot C_p$, and 3-blade propeller $C_n \cdot C_p \cdot C_q$, *arXiv:1812.02880*.

[28] I. Singgih, Subtractive Magic and Antimagic Total Labelings, *arXiv:1811.03033*.

[29] K.A. Sugeng, Magic and Antimagic Labeling of Graphs, *Dissertation*, University of Ballarat, 2005.

[30] D. Yang, J.Carlson, A. Owens, K. E. Perry, I. Singgih, Z-X. Song, F. Zhang, X. Zhang, Antimagic orientations of graphs with large maximum degree, *arXiv:1908.06072*.