Thermodynamics

StatPhys

Test 1

You must show all your work!

(Dated: 02-05-2009)

1. Consider a van der Waals gas with the constant specific heat C_v,

$$P = \frac{NT}{V - Nb} - \frac{N^2 a}{V^2}$$

   (a) Find the expression for the entropy

   (b) Find the heat received, the internal energy change and the work done by the gas in an isothermal expansion from volume V_1 to V_2 at temperature T

   (c) In the case the above expansion is used in a Carnot cycle with two bodies at temperatures $T_2 > T_1$, vdW gas serving as a working body, find the total work done in the cycle

Solution

(a)

$$\begin{aligned} dS &= C_v \frac{dT}{T} + \left(\frac{\partial S}{\partial V}\right)_T dV = C_v \frac{dT}{T} + \left(\frac{\partial P}{\partial T}\right)_V dV \\ &= C_v \frac{dT}{T} + N \frac{dV}{V - Nb} = C_v d\log T + N d\log(V - Nb) \end{aligned}$$

$$S = S_0 + C_v \log \frac{T}{T_0} + N \log \frac{V - Nb}{V_0 - Nb}$$

Notice that

$$\begin{aligned} dE &= TdS - PdV = C_v dT + \left[T\left(\frac{\partial P}{\partial T}\right)_V - P\right] dV \\ &= C_v dT + \frac{N^2 a}{V^2} dV \end{aligned}$$

(b) In isothermal process,

$$\begin{aligned} Q &= T\Delta S = TN \log \frac{V_2 - Nb}{V_1 - Nb} \\ \Delta E &= N^2 a \left(\frac{1}{V_1} - \frac{1}{V_2}\right) \\ R &= N^2 a \left(\frac{1}{V_1} - \frac{1}{V_2}\right) - TN \log \frac{V_2 - Nb}{V_1 - Nb} \end{aligned}$$

(c) In a Carnot cycle,

$$|R| = \frac{T_2 - T_1}{T_2} Q_2 = (T_2 - T_1)\, N \log \frac{V_2 - Nb}{V_1 - Nb}$$

2. Find the adiabatic compressibility $(\partial V/\partial P)_S$ of an ideal gas, $PV = NT$, with the constant specific heat C_v. *Hint*: express $(\partial V/\partial P)_S$ in terms of the isothermal compressibility, $(\partial V/\partial P)_T$.

   Solution

   From (16.15) increases

$$\begin{aligned}\left(\frac{\partial V}{\partial P}\right)_S &= \frac{C_v}{C_p}\left(\frac{\partial V}{\partial P}\right)_T \\ &= -\frac{V^2 C_v}{NT\,(C_v + N)}\end{aligned}$$

3. Ideal gas, $PV = NT$, with the constant specific heat C_v, undergoes an adiabatic expansion wherein its volume increases eightfold. Starting with the same pressure and volume as in the adiabatic process, the gas undergoes an isothermal expansion wherein its final pressure is $2^{11/5}$ times the final pressure and final volume is half the final volume of the adiabatic process. Find the ratio $\gamma = C_p/C_v$.

   Solution

   In adiabatic process

$$\begin{aligned}\frac{P_1}{P_2} &= \left(\frac{V_2}{V_1}\right)^\gamma \\ \frac{P_1}{P_2} &= 8^\gamma = 2^{3\gamma}\end{aligned}$$

   In isothermal process

$$\begin{aligned}\frac{P_1}{2^{11/5} P_2} &= \frac{V_2}{2V_1} \\ \frac{P_1}{P_2} &= 2^{6/5}\frac{V_2}{V_1} = 2^{21/5}\end{aligned}$$

   whereof

$$\gamma = \frac{7}{5}$$