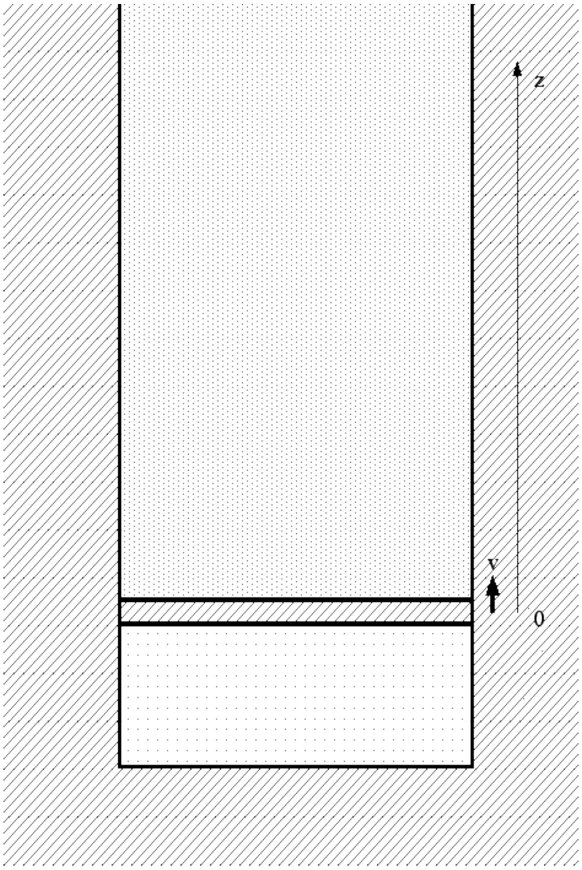
Thermodynamics

Test1

Stat Physics

(Dated: 02-07-2008)



1. An ideal, monatomic gas of N atoms,

   $$PV = NT$$

   with initial volume and pressure V_0 and P_0 respectively, is placed in a thermally isolated shaft. On top, it is separated by a *massless* lid of area A, which is *not* thermally insulating, from a dense gas, whose pressure changes with height as

   $$P = P_0 - \rho g z$$

   (Due to its low density, you may neglect the changes of pressure with height in the ideal gas under the lid. It is also assumed that $z << P_0/\rho g$). The lid is given a very slight push and it starts moving up with a constant velocity v, which is so small that you may consider all parts of the gas under the lid to be in complete equilibrium with each other at any given moment in time.

   After the volume of the gas increases two-fold, and given the constant specific heat of gas,

$$C_v = \frac{3}{2}N$$

find

(a) The total work done by the gas;

(b) Its final temperature;

(c) Total heat received through the lid.

Solution

(a) Since massless lid moves with a constant velocity, pressures on top and bottom are the same. Consequently, the total work done is given by

$$dW = -PdV = -\left(P_0 - \rho g z\right) A dz$$
$$W = -\int_0^h \left(P_0 - \rho g z\right) A dz = -P_0 V_0 + \frac{\rho g A h^2}{2} = -P_0 V_0 + \frac{mgh}{2}$$

where $h = V_0/A$ is the initial height of the container with ideal gas and $m = \rho g V_0$ is the mass of displaced gas on top of the lead.

(b) The energy of the gas

$$E = C_v T = C_v \frac{PV}{N} = \frac{3}{2}PV$$

The total change of energy

$$\triangle E = \frac{3}{2}\left[\left(P_0 - \rho g h\right) 2V_0 - P_0 V_0\right] = \frac{3}{2}\left(P_0 V_0 - 2mgh\right)$$

The final temperature is determined from

$$T = T_0 + \frac{\triangle E}{C_v} = \frac{P_0 V_0}{N} + \frac{\left(P_0 V_0 - 2mgh\right)}{N} = \frac{2\left(P_0 V_0 - mgh\right)}{N}$$

or, alternatively, from

$$N = \frac{\left(P_0 - \rho g h\right) 2V_0}{T}$$
$$T = \frac{2\left(P_0 V_0 - mgh\right)}{N}$$

(c) The total heat received

$$Q = \triangle E - W = \frac{5}{2}P_0 V_0 - \frac{7}{2}mgh$$

2. The volume of a gas is increased two-fold without a change in its internal energy. What will be the change of the temperature in this process in the following two approximations:

   (a) An ideal gas approximation, with constant specific heat C_v;

   (b) A van der Waals approximation

$$P = \frac{NT}{V - Nb} - \frac{N^2 a}{V^2}$$

Solution

For an ideal gas, the temperature does not change since

$$C_v dT = dE = 0$$

For a van der Waals gas,

$$C_v dT = dE - \frac{N^2 a}{V^2} dV = -\frac{N^2 a}{V^2} dV$$

$$\triangle T_{vdW} = \left[\frac{N^2 a}{C_v V} \right]_{V=V_0}^{V=2V_0} = -\frac{N^2 a}{2 C_v V_0}$$