Assignment 1: Thermodynamics

Useful relations: Equations of State of ideal and van der Waals gases

$$P = \frac{NT}{V}$$

$$P = \frac{NT}{V - Nb} - \frac{N^2 a}{V^2}$$

Additional instruction: Derive your results from first principles

1. Show that the Joule-Thompson coefficient $(\partial E/\partial V)_T$ can be expressed in the following form:

$$\left(\frac{\partial E}{\partial V}\right)_T = T\left(\frac{\partial P}{\partial T}\right)_V - P$$

and evaluate it for a van der Waals gas.

2. Consider a gas with arbitrary equation of state $P = f(T, V)$.

a) Calculate $C_P - C_V$ for this gas in terms of f.

b) Using the result of a), calculate $C_P - C_V$ for a van der Waals gas.

3. Consider N molecules of an ideal monatomic gas, $C_V = 3N/2$, placed in a vertical cylinder. The top of the cylinder is closed by a piston of mass M and cross section A. Initially the piston is fixed, and the gas has volume V_0 and temperature T_0. Next, the piston is released, and after several oscillations comes to a stop. Disregarding friction and the heat capacity of the piston and cylinder, find the temperature and volume of the gas at equilibrium. The system is thermally isolated, and the pressure outside the cylinder is P_a.

4. For a van der Waals gas

   a) Prove that $(\partial C_V/\partial V)_T = 0$.

   b) Using a), determine the entropy of the monatomic gas $S\,(T,V)$ and its energy $E\,(T,V)$ to within additive constants.

   Hint: In the limit $V \to \infty$, $C_V = 3N/2$ for a monatomic van der Waals gas.

   c) What is the final temperature when the gas is adiabatically compressed from (V_1, T_1) to V_2. How much work is done in this compression?

5. The operation of a gasoline engine is roughly similar to the Otto cycle:

   $A \to B$ Gas compressed adiabatically, $\triangle S = 0$

   $B \to C$ Gas heated isochorically, $\triangle V = 0$ corresponds to combustion of gasoline)

   $C \to D$ Gas expanded adiabatically (power stroke)

   $D \to A$ Gass cooled isochorically.

   Compute the efficiency of the Otto cycle for an ideal gas as a function of the compression ratio V_A/V_B and the heat capacity C_V.

6. Consider a cylinder of length L with a thin massless piston that divides it into two equal parts. The cylinder is submerged in a large heat bath at temperature T. The left side of the cylinder contains N molecules of ideal gas at pressure P, while the right side is at $P/2$. Let the piston be released.

   a) What is its final equilibrium position?

   b) How much heat will be transmitted to the bath in the process of equilibration?