Quiz 3: Fermi Gas

1. For a d-dimensional, completely degenerate electron gas with dispersion relationship $\varepsilon = p^s$ find

   (a) Total energy E in terms of the Fermi energy ε_F and the total number of particles N

   (b) Pressure P in terms of ε_F and the concentration of particles $n = N/V$

   (c) Relationship between E and PV

   (d) Inverse isothermal compressibility $\kappa_T^{-1} = -V\left(\partial P/\partial V\right)_{N,T=0}$ in terms of ε_F and n

   Solution

$$N = \frac{AV}{s}\int_0^{\varepsilon_F} \varepsilon^{(d-1)/s}\varepsilon^{1/s-1} d\varepsilon = \frac{AV}{s}\int_0^{\varepsilon_F} \varepsilon^{d/s-1} d\varepsilon = \frac{AV}{d}\varepsilon_F^{d/s}$$

$$E = \frac{AV}{s}\int_0^{\varepsilon_F} \varepsilon^{d/s} d\varepsilon = \frac{AV}{d+s}\varepsilon_F^{d/s+1} = \frac{d}{d+s}N\varepsilon_F$$

$$E = B\left[V\left(V^{-s/d}\right)^{d/s+1}\right] = BV^{-s/d}$$

$$P = -\frac{\partial E}{\partial V} = -B\frac{\partial}{\partial V}\left(V^{-s/d}\right) = \frac{s}{d}\frac{E}{V} = \frac{s}{d+s}n\varepsilon_F$$

$$\kappa_T^{-1} = -V\frac{\partial}{\partial V}\left(\frac{s}{d}\frac{E}{V}\right) = -\frac{s}{d}V\frac{\partial}{\partial V}\left(\frac{E}{V}\right) = \frac{s}{d}\left(\frac{E}{V} + P\right)$$
$$= \frac{s}{d}\left(\frac{d}{s} + 1\right)P = \frac{d+s}{d}\frac{s}{d+s}n\varepsilon_F = \frac{s}{d}n\varepsilon_F$$