Quiz 1: Thermodynamics

Equation of state of an ideal gas

$$P = \frac{NT}{V}$$

1. Temperature of an ideal gas with constant C_V, initially at volume V_1, is raised from T_1 to T_2 according to

$$V = \frac{V_1}{T_1} T$$

For this process find

(a) change of entropy

(b) work done by the gas

(c) change of energy

(d) heat transferred to the gas

Solution

$$P = \frac{T}{NV} = \frac{T_1}{NV_1} = P_1 \text{ - isobaric process}$$

Change of entropy

$$\triangle S = C_v \ln\left(\frac{T_2}{T_1}\right) + N \ln\left(\frac{V_2}{V_1}\right) = C_p \ln\left(\frac{T_2}{T_1}\right)$$

Work

$$|R| = P\left(V_2 - V_1\right) = N\left(T_2 - T_1\right)$$

Change of energy

$$\triangle E = C_V (T_2 - T_1)$$

Heat

$$Q = \triangle W = C_p (T_2 - T_1)$$

2. An ideal gas with constant C_V undergoes the following cycle:

- adiabatic expansion from (p_1, V_1) to (p_2, V_2)
- isobaric compression to (p_2, V_1)
- isochoric (constant volume) return to initial state (p_1, V_1)

Find the efficiency of the cycle.

Solution

Work done in the cycle is in adiabatic and isobaric processes (negative)

$$\begin{aligned} |R| &= C_V (T_1 - T_2) - p_2 (V_2 - V_1) \\ &= \frac{C_V}{N} (p_1 V_1 - p_2 V_2) - p_2 (V_2 - V_1) = \frac{1}{\gamma - 1} (p_1 V_1 - p_2 V_2) - p_2 (V_2 - V_1) \end{aligned}$$

Heat received in the cycle is during the isochoric process

$$Q = C_V (T_1 - T_3)$$

Efficiency

$$\begin{aligned} \eta &= \frac{|R|}{Q} = \frac{C_V (T_1 - T_2) - p_2 (V_2 - V_1)}{C_V (T_1 - T_3)} = \frac{(\gamma - 1)^{-1} (p_1 V_1 - p_2 V_2) - p_2 (V_2 - V_1)}{(\gamma - 1)^{-1} (p_1 - p_2) V_1} \\ &= \frac{(p_1 V_1 - p_2 V_2) - (\gamma - 1) p_2 (V_2 - V_1)}{(p_1 - p_2) V_1} = \frac{(p_1 - p_2) V_1 - \gamma p_2 (V_2 - V_1)}{(p_1 - p_2) V_1} \\ &= 1 - \gamma \frac{V_2/V_1 - 1}{p_1/p_2 - 1} \end{aligned}$$

3. A thermally insulated cylinder, closed at both ends, is fitted with a frictionless heat-conducting piston which divides the cylinder into two equal parts containing the same amounts of an ideal gas with constant C_V at temperatures T_1 and T_2 and pressures P_1 and P_2 respectively. The piston is release and the system reaches equilibrium. Find

   (a) the final temperature and pressure

   (b) the change of entropy and show that it is ≥ 0

   Solution

   Since the piston is heat conducting, T will be the same in both compartments. Also P will be the same. Consequently, volumes will be the same and equal.

$$\begin{aligned} 2C_V T &= C_V\left(T_1 + T_2\right) \\ T &= \frac{T_1 + T_2}{2} \\ P &= \frac{NT}{(V/2)} = \frac{P_1 T\,(V/2)}{T_1\,(V/2)} = \frac{P_1 T}{T_1} = P_1 \frac{T_1 + T_2}{2T_1} \\ &\left(\text{also} \;\; = P_2 \frac{T_1 + T_2}{2T_2} \text{ since } \frac{P_1}{T_1} = \frac{P_2}{T_2} = \frac{N}{(V/2)}\right) \end{aligned}$$

   The change of entropy is

$$\triangle S = C_V \ln\left(\frac{T}{T_1}\right) + C_V \ln\left(\frac{T}{T_2}\right) = C_V \ln\left(\frac{T^2}{T_1 T_2}\right) = C_V \ln\left(\frac{\left(T_1 + T_2\right)^2}{4T_1 T_2}\right) \geq 0$$