Assignment 2: Gibbs Distribution

1. Fill in the missing calculations leading to Eq.(30.3) (3 preceding formulae and Eq. (30.3) itself).

2. Use the method described in §36 for Helmholtz free energy F to derive the differential of Ω (as per Eq. (36.3) and notations in the paragraph that follows) and demonstrate that it has the proper thermodynamic form.

3. The mean value of energy of a system in thermal equilibrium is $\overline{E}$.

   a) Show that the energy variance at a constant volume can be described by

   $$\left\langle \left(E - \overline{E}\right)^2 \right\rangle = C_v T^2$$

   b) How does the ratio of rms fluctuation of E to $\overline{E}$ scale with the number of particles in the system?

   c) Write down the normalized Gaussian probability distribution function that would have such a variance.

4. Consider a system composed of $N \gg 1$ distinguishable, non-interacting atoms at rest. Each of the atoms has only two (non-degenerate) energy levels: 0 and $\varepsilon > 0$. Let $\epsilon = E/N$ be the mean energy per atom.

   a) What is the maximum possible value of ϵ?

   b) What is the maximum possible value of ϵ in equilibrium and at what temperature will it occur?

   c) Compute the equilibrium value of entropy $s = S/N$ as a function of ϵ.

5. A zipper has N links; each link has a state in which it is closed with energy 0 and a state in which it is open with energy ε. The zipper can only unzip from one side and that the link can only open if all the preceding links to that side are already open.

a) Find the partition function.

b) Find the average number of open links and investigate your answer in the limit of high and low temperatures.

6. A chain consisting of $N \gg 1$ elliptical links with semi-axes a and b is hanging vertically with its uppermost link attached to a horizontal plane while a mass m is attached to the lowest link.

   a) Find the partition function.

   b) Find the average length of the chain and investigate your answer in the limit of high and low temperatures.