Assignment 3: Ideal Gas

1. A classical system of N distinguishable noninteracting particles of mass m is placed in a 3D harmonic well,

$$U(r) = \frac{x^2 + y^2 + z^2}{2V^{2/3}}$$

a) Find the partition function and the Helmholtz free energy F.

b) Regarding V as an external parameter, find the thermodynamic force $\widetilde{P}$, conjugate to V, exerted by the system; find the equation of state and compare it to that of an ideal gas in a container of volume V.

c) Find the entropy S, internal energy E, and the heat capacity C_V.

2. An ideal gas of particles of mass m at temperature T is placed in an external 1D potential

$$U(x) = Ax^n;\ 0 \leq x \leq \infty,\ A, n > 0$$

Find the average potential energy per particle using two methods:

a) By explicitly evaluating the coordinate part of the partition function;

b) Using the virial and the equipartition theorems.

3. Show *explicitly* that in the limit $T \gg \hbar^2/2I$, eq. (47.3) yields the classical expression for the partition function (leading to eq. (47.10)).

4. The potential energy of a dipole in an electric field is given by

$$U = -\overrightarrow{p} \cdot \overrightarrow{E}$$

Calculate the electric polarization $\overrightarrow{P}$ of an ideal gas of N molecules with electric dipole moment $\overrightarrow{p}$ each at temperature T. Investigate the limits of high and low temperatures. *Hint*: use $\overrightarrow{P} = -\left(\partial F/\partial \overrightarrow{E}\right)$ and choose z-axis along $\overrightarrow{E}$.

5. How much heat is transferred to ideal gas with constant specific heat c_v in compression from V_1 to V_2 in a polytropic process $PV^n = a$ (a constant). Investigate your answer for $n = \gamma$ and $n = 1$. Does the sign of Q change as a function of n?

6. For a cyclic process consisting of two isochoric (V_1 and V_2) and two isothermal (T_1 and T_2) processes find the work done on the ideal gas and the quantity of heat that it gains.

7. Derive eq. (49.6) from eq. (49.1) and eqs. (49.5) and (49.7) from eq. (49.4).