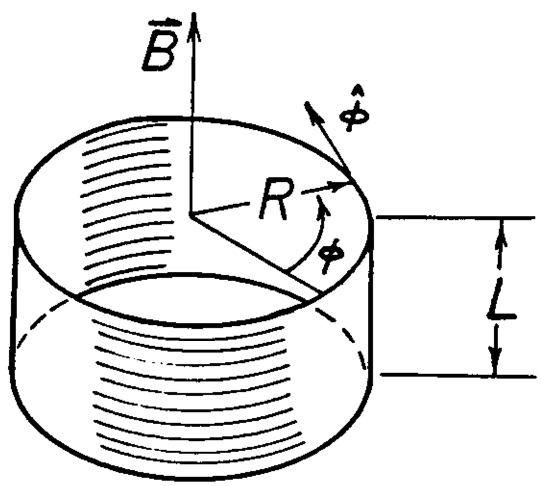
SECTION IV: QUANTUM MECHANICS

IV.1. Flux Quantization



The motion of an electron is confined to the lateral surface of a hollow cylinder of radius R and length L.

a) Write down the wave functions and allowed energies for the electron. You may assume that the appropriate boundary condition is for the wave function to vanish at z = 0 and z = L.

b) Now a weak uniform magnetic field $\vec{B}$ is applied in the z direction. Show that this corresponds to an $\vec{A}$ field given in cylindrical coordinates as

$$\vec{A} = A(r)\ \hat{\phi}$$

where $\hat{\phi}$ is a unit vector. Determine the function A(r), and write down the Schrödinger equation for the electron. Ignore the spin of the electron.

c) Calculate the energy shift of each energy level linear in B using first order perturbation theory.

d) Solve for the eigenvalues exactly in the presence of B and show that it is consistent with the result of part (c).

e) Show by a gauge transformation that the problem in the presence of B is equivalent to the B = 0 problem but with a different boundary condition for the wave function, so that

$$\psi(\phi + 2\pi, z) = \psi(\phi, z)\ e^{i\chi}$$

Determine the function χ.

f) Show that if B takes on special values such that the magnetic flux through the cylinder is an integral multiple of the flux quantum $\Phi_0 = hc/e$, i.e., if

$$B\pi R^2 = n\ \Phi_0,$$

where n is any integer, the eigenvalues are identical.

g) Does the conclusion reached in f) remain true in the presence of an arbitrary potential $V(z, \phi)$?

$\hbar = c = 1$, $\phi_0 = \frac{2\pi}{e}$

$$\left[-\frac{1}{2m}\left(\frac{1}{R}\frac{\partial}{\partial\varphi} - ie\frac{\phi}{2\pi R}\right)^2 + V\right]\psi = E\psi$$

$$\left[-\frac{1}{2MR^2}\left(\frac{\partial}{\partial\varphi} - i\frac{\phi}{\phi_0}\right)^2 + V\right]\psi = E\psi$$

Consider $V = 0$, $\psi = Ae^{im\varphi}$
$m = 0, \pm 1, \pm 2, \ldots$

$$E = \frac{\left(m + \frac{\phi}{\phi_0}\right)^2}{2MR^2}$$

In general, $\psi = \psi_0(\varphi)e^{i\frac{\phi}{\phi_0}\varphi}$

And since $\psi(\varphi + 2\pi) = \psi(\varphi)$,

$$\psi_0(\varphi + 2\pi) = e^{-i\frac{\phi}{\phi_0}2\pi}\psi_0(\varphi)$$

$$\left(-\frac{1}{2MR^2}\frac{\partial^2}{\partial\varphi^2} + V\right)\psi_0 = E\psi_0$$

This is E.V. problem for $\phi = 0$ with b.c.

When $\frac{\phi}{\phi_0} = n$, $\psi_0(\varphi + 2\pi) = \psi_0(\varphi)$

This is E.V. problem identical to that for ψ