QM 15-030-710-003 Spring **
Assignment 9: Identity of Particles, Atom

The due date for this assignment is **.**
Reading assignment: Chapters IX and X.

1. In a system of two identical bosons with spin $s = 0$, one is described by the WF $\psi_1(\mathbf{r})$ while the other by $\psi_2(\mathbf{r})$. Both functions are normalized and have definite - and opposite - parities. Find the coordinate distribution function of one particle given that the position of the other is arbitrary (and not fixed). What is the probability that a) one of the particles and b) both particles are in the half-space $z \geq 0$? Compare your result with the circumstance of distinguishable particles.

2. Solve the problem analogous to the preceding problem for the case of two fermions that are in the same spin state.

3. Two identical spin-zero bosons interact via the potential energy $U = k(\mathbf{r}_1 - \mathbf{r}_2)^2/2$. What is the energy spectrum of the system?

4. A system consists of three identical particles whose coordinates in the centre of mass frame of reference are $\mathbf{r}_1, \mathbf{r}_2$, and $\mathbf{r}_3$ respectively. How does $\mathbf{r}_1 \cdot \mathbf{r}_2$ transform under the permutation of the 1st and 3rd particles? Symmetrize this quantity with respect to permutation of any two particles in the system.

5. Consider the hyperfine structure (HFS) of the s-states of a hydrogenic atom due to the coupling of the magnetic moments of the electron and the nucleus. The nucleus spin is I and the magnetic moment μ_0 so that $\widehat{\boldsymbol{\mu}} = (\mu_0/I)\widehat{\mathbf{I}}$. Estimate the magnitude of the HFS splitting and compare it with the FS splitting.

6. Find the energy and the ionization potential of the ground state of a two-electron ion using the variational principal. As a trial function, take the product of hydrogenic functions with an effective charge Z_{eff} which plays the role of the variational parameter. Based on your result, can you draw any conclusion with respect to existence of a stable H^- ion?

7. Find the normal states of N and Cl atoms.

8. Using the Thomas-Fermi model, find the dependence of the mean distance and mean squared distance of the electron from the nucleus on Z. What is $\overline{r^n}$ for $n \geq 3$? - explain your result.

9. Using the Thomas-Fermi model, find the dependence on Z of the typical value of the orbital angular momentum of the electron and of the energy of full ionization of the atom.