QM 15-030-710-003 Spring ****
Assignment 8: Time Dependence of States

The due date for this assignment is **.**
Reading assignment: Review Chapters II, V, $VIII$ and XV.

1. Find the time dependence of a plane rotator if initially ($t = 0$) it was in a state whose wave function is given by $\Psi(\phi, t = 0) = A \sin^2 \phi$.

   Hint: $\sin^2 \phi = (1 - \cos 2\phi)/2$, where the functions in parentheses are eigenfunctions of the Hamiltonian for plane rotator.

2. Find the time dependence of a spatial rotator if initially ($t = 0$) it was in a state whose wave function is given by $\Psi(\theta, t = 0) = A \cos^2 \theta$.

   Hint: $\cos^2 \theta = \left[1 - \left(1 - 3\cos^2 \theta\right)\right]/3$, where the functions in brackets are eigenfunctions of the Hamiltonian for spatial rotator.

3. Find the velocity $\widehat{\mathbf{v}}$ and acceleration $\widehat{\mathbf{w}}$ operators of a neutral particle with a non-zero magnetic moment (e.g. neutron) in the magnetic field.

4. Find the time dependence of the spin function and the mean values of spin projections of a neutral $s = 1/2$ particle with magnetic moment μ in a spatially uniform magnetic field $\overrightarrow{\mathcal{H}}(t) = \mathcal{H}(t)\,\mathbf{n}_0$. Assume that the initial spin function (and spin projections) are known.

5. A spin $s = 1/2$ particle with magnetic moment μ is subject to a spatially uniform magnetic field $\overrightarrow{\mathcal{H}}(t)$ such that

$$\begin{aligned}\mathcal{H}_x(t) &= \mathcal{H}_0 \cos \omega_0 t \\ \mathcal{H}_y(t) &= \mathcal{H}_0 \sin \omega_0 t \\ \mathcal{H}_z(t) &= \mathcal{H}_1\end{aligned}$$

   Initially ($t = 0$), the particle was in a state with $s_z = 1/2$. Find the probabilities of possible values of s_z at time t. Consider, in particular, the case $|\mathcal{H}_1/\mathcal{H}_0| \ll 1$ and show that "spin flip" for the latter circumstance is of resonance character in terms of dependence on the frequency ω_0. Please, provide a detailed calculation.