QM 15-030-710-002 Spring ****
Assignment 7: Motion in Magnetic Field

The due date for this assignment is **.**
Reading assignment: Chapter XV.

1. Show that in the Coulomb gauge the Hamiltonian of a charged particle in the magnetic field
$$\widehat{H} = \frac{1}{2m}\left(\widehat{\mathbf{p}} - \frac{e}{c}\mathbf{A}\left(\mathbf{r}\right)\right)^2$$
can be written as
$$\widehat{H} = \frac{\widehat{\mathbf{p}}^2}{2m} - \frac{e}{cm}\mathbf{A}\cdot\widehat{\mathbf{p}} + \frac{e^2}{2mc^2}\mathbf{A}^2$$
and check that this operator is Hermitian.

2. Find the velocity operator $\widehat{\mathbf{v}}$ of a charged particle in the magnetic field and establish the commutational relationships $[\widehat{v}_i, \widehat{v}_k]$ and $[\widehat{v}_i, \widehat{x}_k]$.

3. For a charged particle in a constant homogeneous magnetic field, find the operator of the center of the orbit $\widehat{\rho}_0$ of the transverse (perpendicular to the magnetic field) motion and $\widehat{\rho}_0^2$. Also find the operator $\widehat{\rho}_L^2$ of the squared radius of the orbit. Establish the commutational relationships of these operators with each other and with the Hamiltonian.

4. Find the eigenvalue spectrum of $\widehat{\rho}_0^2$ and $\widehat{\rho}_L^2$ (see preceding problem).

5. Find the eigenvalues and eigenfunctions of the stationary states of a neutral $s = 1/2$ particle in a constant homogeneous magnetic field.

6. Find the eigenvalues and eigenfunctions of the stationary states of a charged plane rotator (a charged particle moving in a plane at a fixed distance a form the center) in a constant homogeneous magnetic field perpendicular to the plane.

7. Establish the relationship between the mean values of the orbital angular momentum $\widehat{\mathbf{l}}$ and the magnetic moment $\widehat{\boldsymbol{\mu}}$ of a spin-less charged particle in a magnetic field. Show that this relationship is consistent with gauge invariance.