QM 15-030-710-002 Winter **
Assignment 6: Spin

The due date for this assignment is **.**
Reading assignment: Chapter $VIII$.

1. Solving an eigenvalue/eigenfunction problem for a $s = 1/2$ particle, find the spin functions ψ_{s_i} $(i = 1, 2, 3)$ of the states with definite projections of the spin on x, y and z axes.

2. Find the operator $\widehat{s}_n$ of the spin projection on an arbitrary direction defined by the unit vector $\mathbf{n}$. What is the mean value of the projection on $\mathbf{n}$ in a state with a definite $s_z = 1/2$ (or $-1/2$)? What is the probability to have spin projections $s_n = 1/2$ and $-1/2$ in such states?

3. Find the eigenvalues of the operator $\widehat{f} = a + \mathbf{b}\cdot\widehat{\boldsymbol{\sigma}}$.

4. Simplify the expression $(a\widehat{\boldsymbol{\sigma}})^n$. *Hint*: $\widehat{\sigma}_i\widehat{\sigma}_k = \delta_{ik} + i\varepsilon_{ikl}\widehat{\sigma}_l$.

5. Find the projection operators $P_{s_z=\pm 1/2}$ to the states with definite projection $s_z = \pm 1/2$ on axis z. *Notice*: such operators are Hermitian and satisfy the relationship $P^2_{s_z=\pm 1/2} = P_{s_z=\pm 1/2}$.

6. For a $s = 1/2$ particle, find the transformation law of the spin function $\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$ under rotation of the coordinate system by an angle φ around the axis whose direction is defined by a unit vector $\mathbf{n}$. Show that the quantity $\phi^*\psi = \phi_1^*\psi_1 + \phi_2^*\psi_2$ remains unchanged under such transformation, that is, it is a scalar.

7. Show that, in a state of two particles with a definite value of the total spin, the operator $\widehat{\boldsymbol{\sigma}}_1\cdot\widehat{\boldsymbol{\sigma}}_2$ also takes a definite value.

8. Using the result of the preceding problem, find the projection operators $P_{S=0,1}$ to the states of a two-particle system, each with spin $s = 1/2$, with definite values of the total spin.

9. Find eigenfunctions and eigenvalues of the operator $a\,(\widehat{\sigma}_{1z} + \widehat{\sigma}_{2z}) + b\widehat{\boldsymbol{\sigma}}_1\cdot\widehat{\boldsymbol{\sigma}}_2$.

10. In a state of a $s = 1/2$ particle with definite values l, m, s_z, find the probabilities of different values of $\mathbf{j} = \mathbf{l} + \mathbf{s}$.

11. Helicity is defined as a projection of a spin to the direction of the momentum. For a a $s = 1/2$ particle, find the wavefunctions $\psi_{\mathbf{p},\lambda}$ of the states with definite momentum $\mathbf{p}$ and helicity $\lambda = \pm 1/2$.

12. Find the form of the helicity operator and show that it commutes with $\widehat{\mathbf{j}} = \widehat{\mathbf{l}} + \widehat{\mathbf{s}}$.