

Assignment 6: Spin

The due date for this assignment is ****.

Reading assignment: Chapter VIII.

1. Solving an eigenvalue/eigenfunction problem for a $s = 1/2$ particle, find the spin functions ψ_{s_i} ($i = 1, 2, 3$) of the states with definite projections of the spin on x , y and z axes.
2. Find the operator \hat{s}_n of the spin projection on an arbitrary direction defined by the unit vector \mathbf{n} . What is the mean value of the projection on \mathbf{n} in a state with a definite $s_z = 1/2$ (or $-1/2$)? What is the probability to have spin projections $s_n = 1/2$ and $-1/2$ in such states?
3. Find the eigenvalues of the operator $\hat{f} = a + \mathbf{b} \cdot \hat{\boldsymbol{\sigma}}$.
4. Simplify the expression $(a\hat{\boldsymbol{\sigma}})^n$. *Hint:* $\hat{\sigma}_i\hat{\sigma}_k = \delta_{ik} + i\varepsilon_{ikl}\hat{\sigma}_l$.
5. Find the projection operators $P_{s_z=\pm 1/2}$ to the states with definite projection $s_z = \pm 1/2$ on axis z . *Notice:* such operators are Hermitian and satisfy the relationship $P_{s_z=\pm 1/2}^2 = P_{s_z=\pm 1/2}$.
6. For a $s = 1/2$ particle, find the transformation law of the spin function $\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$ under rotation of the coordinate system by an angle φ around the axis whose direction is defined by a unit vector \mathbf{n} . Show that the quantity $\phi^*\psi = \phi_1^*\psi_1 + \phi_2^*\psi_2$ remains unchanged under such transformation, that is, it is a scalar.
7. Show that, in a state of two particles with a definite value of the total spin, the operator $\hat{\boldsymbol{\sigma}}_1 \cdot \hat{\boldsymbol{\sigma}}_2$ also takes a definite value.
8. Using the result of the preceding problem, find the projection operators $P_{S=0,1}$ to the states of a two-particle system, each with spin $s = 1/2$, with definite values of the total spin.
9. Find eigenfunctions and eigenvalues of the operator $a(\hat{\sigma}_{1z} + \hat{\sigma}_{2z}) + b\hat{\boldsymbol{\sigma}}_1 \cdot \hat{\boldsymbol{\sigma}}_2$.
10. In a state of a $s = 1/2$ particle with definite values l, m, s_z , find the probabilities of different values of $\mathbf{j} = \mathbf{l} + \mathbf{s}$.
11. Helicity is defined as a projection of a spin to the direction of the momentum. For a $s = 1/2$ particle, find the wavefunctions $\psi_{\mathbf{p},\lambda}$ of the states with definite momentum \mathbf{p} and helicity $\lambda = \pm 1/2$.
12. Find the form of the helicity operator and show that it commutes with $\hat{\mathbf{j}} = \hat{\mathbf{l}} + \hat{\mathbf{s}}$.