QM 15-030-710-003 Winter ****
Assignment 5: Perturbation Theory

The due date for this assignment is **.**
Reading assignment: Chapter VI.

1. For a particle in an infinitely deep potential well of width a $(0 < x < a)$, find the first-order corrections to the energy levels under the following perturbations:

   a)
   $$V(x) = \frac{V_0}{a}\left(a - |2x - a|\right)$$

   b)
   $$V(x) = \left\{ \begin{matrix} V_0,\ b < x < a - b \\ 0,\ 0 < x < b \text{ and } a - b < x < a \end{matrix} \right\}$$

   Analyze the conditions of applicability of the perturbation theory.

2. Show that for an arbitrary perturbation $V(x)$, the first order correction $E_n^{(1)}$ to the energy levels in the potential of Problem 1 does not depend on n for sufficiently large n. Analyze the conditions of applicability of the perturbation theory.

3. A charged linear harmonic oscillator is placed in the uniform electric field $\mathcal{E}$ directed along the axis of oscillations. Treating the electric field as a perturbation, evaluate the energy level corrections to second order and compare your result with the exact solution.

4. A plane rotator with the moment of inertia I and the dipolar moment $\mathbf{d}$ is placed in a uniform electric field $\vec{\mathcal{E}}_0$ (directed in the plane of rotation) which can be treated perturbatively. Find the polarizability of the rotator's ground state.

   Hint: $V = -d\mathcal{E}_0 \cos\phi = -d\mathcal{E}_0 \left[\exp(i\phi) + \exp(-i\phi)\right]/2$.

5. At $t = -\infty$, a particle in the ground state of the potential of Problem 1 is a subject to a weak, time-dependent perturbation of the following form:

   a) $V(x,t) = -xF_0 \exp\left(-t^2/\tau^2\right)$
   b)$V(x,t) = -xF_0 \exp\left(-|t|/\tau\right)$
   c)$V(x,t) = -xF_0 / \left(1 + t^2/\tau^2\right)$

   Using perturbation theory to the first order, evaluate the probabilities of transitions to other eigenstates at $t \to \infty$.

6. A plane rotator with the dipole moment $\mathbf{d}$ is placed in a spatially uniform variable electric field $\vec{\mathcal{E}}(t)$, $\mathcal{E}(t) = f(t)\,\mathcal{E}_0$. Before the field was turned on, the rotator had a definite value of the projection of the angular momentum m. Using the first-order perturbation theory, evaluate the probabilities of different values of the projection and energy at $t \to \infty$. Consider a particular case

   $$f(t) = \left\{ \begin{matrix} 0,\ t < 0 \\ \exp(-t/\tau),\ t > 0 \end{matrix} \right\}$$

7. A charged linear harmonic oscillator is subject to a spatially uniform electric field $\mathcal{E}(t) = \mathcal{E}_0 / \left(1 + t^2/\tau^2\right)$. Before the field was turned on, the oscillator was in the n-th stationary state. The total impulse of the force is P_0 . In the first order of the perturbation theory, find the probabilities of excitation to other possible states. Analyze the limiting cases $\omega\tau \ll 1$ and $\omega\tau \gg 1$.