QM 15-Phys-710
Fall 2000
Quiz 1: Operators In Quantum Mechanics
Friday, October 13

1. $\widehat{L}$ is a linear operator. Is operator $\widehat{L}\widehat{L}^{\dagger}$ Hermitian? - explain your answer.

$$\left(\widehat{L}\widehat{L}^{\dagger}\right)^{\dagger} = \left(\widehat{L}^{\dagger}\right)^{\dagger}\widehat{L}^{\dagger} = \widehat{L}\widehat{L}^{\dagger} \text{ - Hermitian}$$

2. $\widehat{f}$ and $\widehat{g}$ are non-commuting Hermitian operators. Are the following operators Hermitian? - explain your answer.

   a) $\widehat{g}\widehat{f}\widehat{g}$

$$\left(\widehat{g}\widehat{f}\widehat{g}\right)^{\dagger} = \widehat{g}^{\dagger}\widehat{f}^{\dagger}\widehat{g}^{\dagger} = \widehat{g}\widehat{f}\widehat{g} \text{ - Hermitian}$$

   b) $\widehat{f}\widehat{g} - \widehat{g}\widehat{f}$

$$\left(\widehat{f}\widehat{g} - \widehat{g}\widehat{f}\right)^{\dagger} = \widehat{g}^{\dagger}\widehat{f}^{\dagger} - \widehat{f}^{\dagger}\widehat{g}^{\dagger} = \widehat{g}\widehat{f} - \widehat{f}\widehat{g} = -\left(\widehat{f}\widehat{g} - \widehat{g}\widehat{f}\right) \text{ - anti-Hermitian}$$

3. Find the eigenvalues of a Hermitian operator $\widehat{f}$ such that $\widehat{f}^2 = c\widehat{f}$, where c is real.

$$\begin{aligned} f_i^2 &= cf_i \\ f_{i1} &= 0,\ f_{i2} = c \end{aligned}$$

4. $F\left(x, x^{'}\right)$ is the kernel of the operator $\widehat{f}$ and $F_{inv}\left(x, x^{'}\right)$ is the kernel of the operator $\widehat{f}^{-1}$. Find

$$\int F\left(x, x^{'}\right) F_{inv}\left(x^{'}, x^{''}\right) dx^{'}$$

   This is the kernel of the unit operator $\widehat{f}\widehat{f}^{-1}$. Therefore,

$$\int F\left(x, x^{'}\right) F_{inv}\left(x^{'}, x^{''}\right) dx^{'} = \delta\left(x - x^{''}\right)$$

5. For the operator

$$\widehat{l}_z = i\left(y\frac{\partial}{\partial x} - x\frac{\partial}{\partial y}\right)$$

   what is $\widetilde{\widehat{l}_z}$, $\widehat{l}_z^{*}$, and $\widehat{l}_z^{\dagger}$?

$$\begin{aligned} \widetilde{\widehat{l}_z} &= \widehat{l}_z^{*} = -\widehat{l}_z \\ \widehat{l}_z^{\dagger} &= \widehat{l}_z \end{aligned}$$

6. The inversion operator is defined as

$$\widehat{I}\Psi(\mathbf{r}) = \Psi(-\mathbf{r})$$

   For the operator $\widehat{l}_z$ from the preceding problem, find the commutator $\left[\widehat{I}, \widehat{l}_z\right]$.

   Since all coordinates change sign under inversion, operator $\widehat{l}_z$ doesn't and $\left[\widehat{I}, \widehat{l}_z\right] = 0$.