

QM 15-Phys-710
Fall 2000
Quiz 1: Operators In Quantum Mechanics
Friday, October 13

1. \widehat{L} is a linear operator. Is operator $\widehat{L}\widehat{L}^\dagger$ Hermitian? - explain your answer.

$$\left(\widehat{L}\widehat{L}^\dagger\right)^\dagger = \left(\widehat{L}^\dagger\right)^\dagger \widehat{L}^\dagger = \widehat{L}\widehat{L}^\dagger - \text{Hermitian}$$

2. \widehat{f} and \widehat{g} are non-commuting Hermitian operators. Are the following operators Hermitian? - explain your answer.

a) $\widehat{g}\widehat{f}\widehat{g}$

$$\left(\widehat{g}\widehat{f}\widehat{g}\right)^\dagger = \widehat{g}^\dagger \widehat{f}^\dagger \widehat{g}^\dagger = \widehat{g}\widehat{f}\widehat{g} - \text{Hermitian}$$

b) $\widehat{f}\widehat{g} - \widehat{g}\widehat{f}$

$$\left(\widehat{f}\widehat{g} - \widehat{g}\widehat{f}\right)^\dagger = \widehat{g}^\dagger \widehat{f}^\dagger - \widehat{f}^\dagger \widehat{g}^\dagger = \widehat{g}\widehat{f} - \widehat{f}\widehat{g} = -\left(\widehat{f}\widehat{g} - \widehat{g}\widehat{f}\right) - \text{anti-Hermitian}$$

3. Find the eigenvalues of a Hermitian operator \widehat{f} such that $\widehat{f}^2 = c\widehat{f}$, where c is real.

$$\begin{aligned} f_i^2 &= cf_i \\ f_{i1} &= 0, f_{i2} = c \end{aligned}$$

4. $F(x, x')$ is the kernel of the operator \widehat{f} and $F_{inv}(x, x')$ is the kernel of the operator \widehat{f}^{-1} . Find

$$\int F(x, x') F_{inv}(x', x'') dx'$$

This is the kernel of the unit operator $\widehat{f}\widehat{f}^{-1}$. Therefore,

$$\int F(x, x') F_{inv}(x', x'') dx' = \delta(x - x'')$$

5. For the operator

$$\widehat{l}_z = i\left(y\frac{\partial}{\partial x} - x\frac{\partial}{\partial y}\right)$$

what is \widetilde{l}_z , \widehat{l}_z^* , and \widehat{l}_z^\dagger ?

$$\begin{aligned} \widetilde{l}_z &= \widehat{l}_z^* = -\widehat{l}_z \\ \widehat{l}_z^\dagger &= \widehat{l}_z \end{aligned}$$

6. The inversion operator is defined as

$$\widehat{I}\Psi(\mathbf{r}) = \Psi(-\mathbf{r})$$

For the operator \widehat{l}_z from the preceding problem, find the commutator $[\widehat{I}, \widehat{l}_z]$.

Since all coordinates change sign under inversion, operator \widehat{l}_z doesn't and $[\widehat{I}, \widehat{l}_z] = 0$.