QM 15-Phys-710-001(/002/003) Fall 1999
Quiz 2: One-Dimensional Motion
Monday, November 8

1. Using Heisenberg's uncertainty relations, determine the lower limit of the possible values of the energy of a particle moving in a field of potential energy

$$V(x) = A\left(\exp(-2\alpha x) - 2\exp(-\alpha x)\right)$$

Compare you result with the ground state energy given that the exact spectrum is as follows:

$$E_n = -A\left[1 - \frac{\alpha\hbar}{\sqrt{2mA}}\left(n + \frac{1}{2}\right)\right]^2$$

(Where n is limited by the condition that the expression inside the square brackets remains non-negative). You may assume that

$$A \gg \frac{\hbar^2\alpha^2}{2m}$$

Solution

$$E_0 = -A\left[1 - \frac{\alpha\hbar}{2\sqrt{2mA}}\right]^2 \simeq -A + \sqrt{\frac{\hbar^2\alpha^2}{2m}A}$$

On the other hand,

$$V(x) = A\left(\exp(-2\alpha x) - 2\exp(-\alpha x)\right) \simeq -A + A\left(\frac{(-2\alpha x)^2}{2} - 2\frac{(-\alpha x)^2}{2}\right) = -A + A\alpha^2x^2$$

And

$$\overline{E} = -A + A\alpha^2\overline{x^2} + \frac{\overline{p^2}}{2m} \geq -A + A\alpha^2\delta x^2 + \frac{\delta p^2}{2m} \geq -A + A\alpha^2\delta x^2 + \frac{\hbar^2}{8m\delta x^2}$$

Minimizing with respect to δx^2,

$$\delta x^2_{\text{min}} = \sqrt{\frac{\hbar^2}{8mA\alpha^2}}$$

So

$$\overline{E} \geq -A + 2A\alpha^2\sqrt{\frac{\hbar^2}{8mA\alpha^2}} = -A + \sqrt{\frac{\hbar^2\alpha^2}{2m}A}$$