QM 15-030-710-001 Fall 1998
Final Exam
Wednesday, December 9

$(11 + 11 + 11 + 12$ points)

1. Consider a shallow
$$U_0 \ll \frac{\hbar^2}{ma^2}$$
symmetrical potential well
$$U(x) = \begin{matrix} -U_0, |x| < a \\ 0, \quad |x| > a \end{matrix}$$
Obtain the approximate expressions for the energy and the normalized wave function of the discrete spectrum level in this well and determine the probability of finding the particle inside the well.

2. For a potential profile
$$U(x) = \begin{matrix} U_0 > 0, x > 0 \\ 0, \quad x < 0 \end{matrix}$$
find the wave functions (up to a normalization constant) of the stationary states with energies $E < U_0$. Do these states form a complete set? What are the transmission and reflection coefficients for such states?

3. Find the following commutator:
$$\left[\widehat{l}_i, a\widehat{\mathbf{r}} + b\widehat{\mathbf{p}}\right]$$
Specifically, consider the case $i = 1$ ($i = x$).

4. In a state ψ_{lm} with definite values of the angular monument l and its projection m along the z-axis, find the mean values $\overline{\widehat{l}_x^2}$ and $\overline{\widehat{l}_y^2}$.