Mechanics, Spring 2001
Final
Monday, May 7

1. (20) Find the eigenvibrations of the system depicted in the Figure if at $t = 0$ one of the particles moves with velocity v while the second particle is at rest and the displacement from the equilibrium position of both particles is zero.

   Solution

$$\begin{aligned}\omega_1 &= \omega_0 = \sqrt{\frac{\varkappa}{m}} \\ \omega_2 &= \sqrt{3}\omega_0\end{aligned}$$

and

$$\begin{aligned}x_1 &= A\cos(\omega_1 t + \phi_1) + B\cos(\omega_2 t + \phi_2) \\ x_2 &= A\cos(\omega_1 t + \phi_1) - B\cos(\omega_2 t + \phi_2)\end{aligned}$$

From the initial conditions

$$\begin{aligned}A\cos(\phi_1) &= 0 \\ B\cos(\phi_2) &= 0 \\ -2A\omega_1\sin(\phi_1) &= v \\ -2B\omega_2\sin(\phi_2) &= v\end{aligned}$$

whereof

$$\begin{aligned}\phi_1 &= \phi_2 = \frac{\pi}{2} \\ -2A\omega_1 &= -2B\omega_2 = v\end{aligned}$$

and

$$\begin{aligned}x_1 &= \frac{v}{2\omega_1}\sin(\omega_1 t) + \frac{v}{2\omega_2}\sin(\omega_2 t) \\ x_2 &= \frac{v}{2\omega_1}\sin(\omega_1 t) - \frac{v}{2\omega_2}\sin(\omega_2 t)\end{aligned}$$

2. (10) Consider precession of a symmetrical top about the direction of the constant angular momentum M (see Figure). Show that

$$\tan(\theta) = \frac{I_1}{I_3}\tan(\alpha)$$

   Solution

$$\begin{aligned}M_1 &= M\sin(\theta) \\ M_3 &= M\cos(\theta)\end{aligned}$$

so that

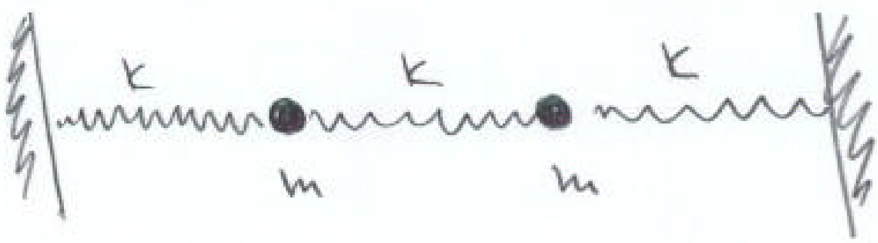
$$\tan(\theta) = \frac{M_1}{M_3}$$

On the other hand,

$$\begin{aligned}M_1 &= I_1\Omega_1 = I_1\Omega\sin(\alpha) \\ M_3 &= I_3\Omega_3 = I_3\Omega\cos(\alpha)\end{aligned}$$

whereof

$$\frac{M_1}{M_3} = \frac{I_1}{I_3}\tan(\alpha)$$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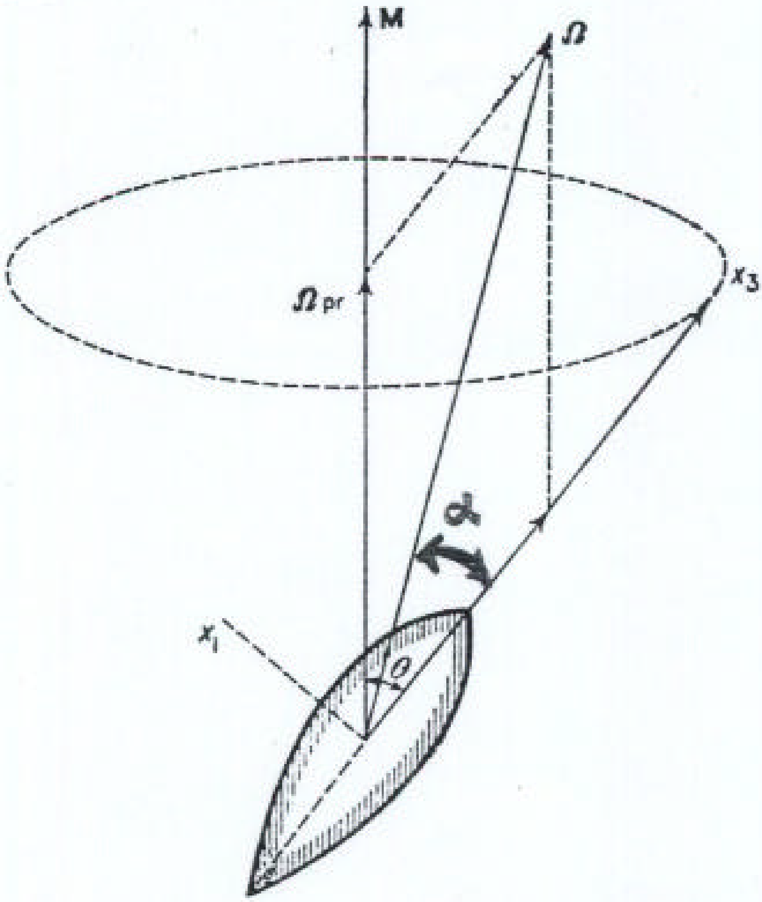
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