MT 13-20

Consider eq. (13.31). On the l.h.s. m/d ordinarily goes to ρ. However, if we have a concentrated mass, we, instead, cancel d in m/d on the l.h.s. and τ/d on the r.h.s. Then we take the limit of $d \to 0$ and, as a result, obtain the discontinuity in the derivative, namely

$$\frac{\partial q}{\partial x}\,|_{0+} - \frac{\partial q}{\partial x}\,|_{0-} = \frac{m}{\tau}\,\ddot{q}|_0$$

On the other hand, per(13.33),

$$\ddot{q} = \frac{\rho}{\tau}\frac{\partial^2 q}{\partial x^2} = v^2 \frac{\partial^2 q}{\partial x^2}$$

Going to the notations of §13.7, we find solution the wave equation

$$\frac{\partial^2 \psi}{\partial x^2} + k^2 \psi = 0,\, k^2 = \frac{\omega^2}{v^2}$$

subject to the boundary conditions

$$\begin{aligned} \psi_2(0) &= \psi_1(0) \\ \frac{\partial \psi_2}{\partial x} \,|&_{x=0} - \frac{\partial \psi_1}{\partial x}\,|_{x=0} = -\frac{M}{\tau} v^2 k^2 \psi(0) = -\frac{M\omega^2}{\tau}\psi(0) \end{aligned}$$

(notice that per first b.c. it does not matter whether we use $\psi_2(0)$ or $\psi_1(0)$ on the r.h.s. of the second), where

$$\psi_1(x) = \exp(ikx) + B\exp(-ikx)$$

is the incident and reflected waves and

$$\psi_2(x) = C\exp(ikx)$$

is the outgoing wave. The reflection and transmission coefficients are $R = |B|^2$ and $T = |C|^2$ respectively.