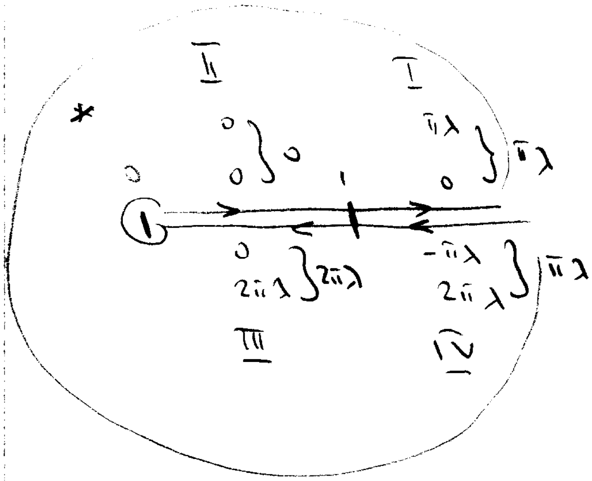

$\theta_1 = \arg t \Rightarrow \arg t^\lambda = \lambda\theta_1$

$\theta_2 = \arg(1-t) \Rightarrow \arg(1-t)^{-\lambda} = -\lambda\theta_2$

I : $\theta_1 = 0$, $\theta_2 = -\pi$

II : $\theta_1 = 0$, $\theta_2 = 0$

III : $\theta_1 = 2\pi$, $\theta_2 = 0$

IV : $\theta_1 = 2\pi$, $\theta_2 = \pi$

$$\int_0^1 \left(\frac{t}{1-t}\right)^\lambda \frac{dt}{t-z} + \int_1^0 \left(\frac{t}{1-t}\right)^\lambda \frac{dt}{t-z} e^{2\pi i\lambda} + \underbrace{\int \left(\frac{t}{1-t}\right)^\lambda \frac{dt}{t-z}}_{\text{large circle}} = 2\pi i \left(\frac{z}{1-z}\right)^\lambda \quad (-1)^\lambda = e^{i\pi\lambda}$$

$$\left(1-e^{2\pi i\lambda}\right) I + \underbrace{\int \frac{dt}{t-z}}_{\text{closed contour}} = 2\pi i e^{\pi i\lambda} \left(\frac{z}{z-1}\right)^\lambda$$

$$\cancel{e^{\pi i\lambda}} \left(-2i\sin\pi\lambda\right) I + 2\pi i \cancel{e^{i\pi\lambda}} = 2\pi i\, \cancel{e^{\pi i\lambda}} \left(\frac{z}{z-1}\right)^\lambda$$

$$\sin\pi\lambda\, I = \pi\left[1 - \left(\frac{z}{z-1}\right)^\lambda\right], \quad I = \frac{\pi}{\sin\pi\lambda}\left[1 - \left(\frac{z}{z-1}\right)^\lambda\right]$$

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if $z \in (0,1)$

$$e^{\pi i\lambda} I \left(-2i\sin\pi\lambda\right) + 2\pi i e^{i\pi\lambda} = \pi i \left[\left(\frac{z}{1-z}\right)^\lambda + e^{2\pi i\lambda}\left(\frac{z}{1-z}\right)^\lambda\right]$$

$$= \pi i e^{i\pi\lambda}\, 2\cos\lambda\pi \Rightarrow I = \frac{\pi}{\sin\pi\lambda}\left[1 - \left(\frac{z}{1-z}\right)^\lambda \cos\pi\lambda\right]$$