$$\left(\frac{\partial}{\partial t} - D\nabla^2\right) G(\vec{r},t) = \delta(\vec{r},t) \equiv \delta(\vec{r})\delta(t)$$

$$\left(\frac{\partial}{\partial t} + Dk^2\right) G(\vec{k},t) = \frac{1}{(2\pi)^d}\delta(t)$$

$$G(\vec{k},t) = \frac{1}{(2\pi)^d} e^{-Dk^2 t}\,\theta(t)$$

$$G(\vec{r},t) = \frac{\theta(t)}{(2\pi)^d}\int d^d\vec{k}\, e^{-Dk^2 t} e^{i\vec{k}\cdot\vec{r}}$$

$$= \frac{\theta(t)}{(2\pi)^d}\prod_{i=1}^{d}\int dk_i\, e^{-Dk_i^2 t + ik_i x_i}$$

$$= \frac{\theta(t)}{(2\pi)^d}\prod_{i=1}^{d}\left[\int dk_i\; e^{-Dt\left(k_i - \frac{ix_i}{2Dt}\right)^2} e^{-\frac{x_i^2}{4Dt}}\right]$$

$$= \frac{\theta(t)}{(2\pi)^d}\left(\frac{\pi}{Dt}\right)^{d/2} e^{-\frac{r^2}{4Dt}}$$

$$= \theta(t)\left(\frac{1}{\sqrt{4Dt\pi}}\right)^d e^{-\frac{r^2}{4Dt}}$$

Green's function of diffusion operator