$$u_{tt} + \sigma u_t - c^2 u_{xx} = f(x,t)$$

$$G_{tt} + \sigma G_t - c^2 G_{xx} = \delta(x)\delta(t) \equiv \delta(x,t)$$

$$G(t,x) = \int_{-\infty}^{\infty} d\omega\, G(\omega,x) e^{i\omega t}, \quad \delta(x,t) = \delta(x) \frac{1}{2\pi}\int_{-\infty} d\omega\, e^{i\omega}$$

$$\left[-\omega^2 + i\sigma\omega - c^2 \partial_{xx}\right] G(\omega,x) = \frac{1}{2\pi}\delta(x)$$

$$-\omega^2 + i\sigma\omega \equiv -k^2c^2$$

$$\left[\partial_{xx} + k^2\right] G(\omega,x) = -\frac{1}{2\pi c^2}\delta(x)$$

$$G(\omega,x) = -\frac{2}{2\pi \ell c^2}\sum_n \frac{\sin\frac{n\pi x}{\ell}\sin\frac{n\pi x'}{\ell}}{k^2 - \left(\frac{n\pi}{\ell}\right)^2}$$

$$= -\frac{1}{\pi \ell c^2}\sum_n \frac{\sin\frac{n\pi x}{\ell}\sin\frac{n\pi x'}{\ell}}{\left(\frac{\omega}{c}\right)^2 - i\frac{\sigma\omega}{c^2} - \left(\frac{n\pi}{\ell}\right)^2}$$

$$= -\frac{1}{\pi \ell c^2}\sum_n \frac{\sin\frac{n\pi x}{\ell}\sin\frac{n\pi x'}{\ell}}{\left(\frac{\omega}{c} - \frac{i}{2}\frac{\sigma}{c}\right)^2 + \left(\frac{\sigma}{2c}\right)^2 - \left(\frac{n\pi}{\ell}\right)^2}$$

Assume $\frac{\pi}{\ell} > \frac{\sigma}{2c}$, $\left(\frac{\omega_n}{c}\right)^2 \equiv \left(\frac{n\pi}{\ell}\right)^2 - \left(\frac{\sigma}{2c}\right)^2$

$$= -\frac{1}{\pi \ell}\sum_n \frac{\sin\frac{n\pi x}{\ell}\sin\frac{n\pi x'}{\ell}}{\left(\omega - \omega_n - i\frac{\sigma}{2}\right)\left(\omega + \omega_n - i\frac{\sigma}{2}\right)}$$

$$G(t,x) = \int_{-\infty}^{\infty} d\omega\, G(\omega,x) e^{i\omega t}$$

$$= -\frac{1}{\pi l} \sum_n \cdots \sin \cdots \int_{-\infty} \frac{\qquad}{\left(\omega - \omega_n - i\frac{\sigma}{2}\right)\left(\omega + \omega_n - i\frac{\sigma}{2}\right)}$$

$$= -\frac{1}{\pi l}\theta(t) \sum_n \sin\frac{n\pi x}{l} \sin\frac{n\pi x'}{l}\, 2\pi i \left[\frac{e^{i(\omega_n + \frac{i}{2}\sigma)t}}{2\omega_n} + \frac{e^{i(-\omega_n + \frac{i}{2}\sigma)t}}{-2\omega_n} \right]$$

$$= -\frac{1}{\pi l}\theta(t) \sum_n \sin\frac{n\pi x}{l} \sin\frac{n\pi x'}{l} e^{-\frac{\sigma}{2}t}\, 2\pi i \;\frac{2i \sin\omega_n t}{2\omega_n}$$

$$= \frac{2}{l}\theta(t) e^{-\frac{\sigma}{2}t} \sum_n \sin\frac{n\pi x}{l} \sin\frac{n\pi x'}{l} \;\frac{\sin\omega_n t}{\omega_n}$$

Notice that $\left.\frac{\partial G}{\partial t}\right|_{t=0} = 0$

Consider $\frac{\sigma}{2c} \ll \frac{\pi}{l}$

$$\simeq \frac{2}{\cancel{l}}\theta(t) e^{-\frac{\sigma}{2}t} \sum_n \sin\frac{n\pi x}{l} \sin\frac{n\pi x'}{l} \;\frac{\sin n\pi\frac{ct}{l}}{c\frac{n\pi}{\cancel{l}}}$$

$$= \frac{2}{\pi c}\theta(t) e^{-\frac{\sigma}{2}t} \sum_n \sin\frac{n\pi x}{l} \sin\frac{n\pi x'}{l} \sin\frac{n\pi ct}{l} \frac{1}{n}$$

(*) Or multiply by $e^{-i\omega t}$, take integral on t from $-\infty$ to ∞ and integrate by parts

$$-\omega^2 \int_{-\infty}^{\infty} G(t,x)\, e^{-i\omega t} + i\omega\sigma \int_{-\infty}^{\infty} G(t,x)\, e^{-i\omega t}$$

$$- c^2 \partial_{xx} \int_{-\infty}^{\infty} G(t,x) e^{-i\omega t} = \delta(x)$$

$$G(\omega,x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} G(t,x)\, e^{-i\omega t}\, dt$$

$$\left(-\omega^2 + i\omega\sigma - c^2\partial_{xx}\right) 2\pi\, G(\omega,x) = \delta(x)$$