

2D Green's fn. using Sturm-Liouville (compare pp 271-272)

$$(\nabla^2 + k^2) G(r, r'; \theta) = \frac{1}{r} \delta(r-r') \delta(\theta), \quad -\pi \leq \theta \leq \pi$$

$$\delta(\theta) = \frac{1}{\pi} \sum_m \frac{1}{\epsilon_m} \cos m\theta$$

$$G(r, r'; \theta) = \sum_m G_m(r, r') \cos m\theta$$

$$\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial G_m}{\partial r} \right) - \frac{m^2}{r^2} G_m + k^2 G_m = \frac{1}{r} \delta(r-r') \frac{1}{\pi}$$

$$\frac{\partial}{\partial r} \left( r \frac{\partial G_m}{\partial r} \right) + \left( k^2 r - \frac{m^2}{r} \right) G_m = \delta(r-r') \frac{1}{\pi \epsilon_m}$$

$$p = -r, \quad q = k^2 r - \frac{m^2}{r}$$

$$v_1 = J_m(kr), \quad v_2 = J_m(kr) Y_m(kR) - J_m(kR) Y_m(kr)$$

$$w = \begin{vmatrix} J_m(kr) & J_m(kr) Y_m(kR) - J_m(kR) Y_m(kr) \\ k J_m'(kr) & k (J_m'(kr) Y_m(kR) - J_m(kR) Y_m'(kr)) \end{vmatrix}$$

$$= -k (J_m(kr) Y_m'(kr) - J_m'(kr) Y_m(kr)) J_m(kR)$$

$$= -\frac{2}{\pi r} J_m(kR)$$

$$G_m(r, r') = -\frac{1}{2\epsilon_m J_m(kR)} \begin{cases} J_m(kr) [J_m(kr) Y_m(kR) - J_m(kR) Y_m(kr)], & r < r' \\ J_m(kr') [J_m(kr) Y_m(kR) - J_m(kR) Y_m(kr)], & r > r' \end{cases}$$