$$(z+\sqrt{z^2-1}\cos\phi)^n = \rho^n$$

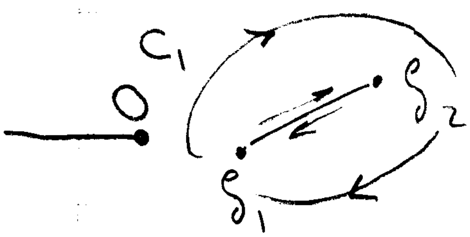
$$d\phi\,\sin\phi = \frac{d\rho}{\sqrt{z^2-1}}, \quad d\phi = \frac{d\rho}{\sin\phi\sqrt{z^2-1}}$$

$$\sin\phi = \sqrt{1-\cos^2\phi} = \sqrt{1-\frac{(\rho-z)^2}{z^2-1}} = \frac{i\sqrt{1-2\rho z+\rho^2}}{\sqrt{z^2-1}}$$

$$d\phi = \frac{1}{i}\,\frac{d\rho}{\sqrt{1-2\rho z+\rho^2}}$$

$$P_n(z) = \frac{1}{2\pi}\int_0^{2\pi}(z+\sqrt{z^2-1}\cos\phi)^n\,d\phi$$

$$= \frac{1}{2\pi i}\int_C \frac{d\rho\,\rho^n}{\sqrt{1-2\rho z+\rho^2}} = \frac{1}{2\pi i}\int_{C_1}$$



$$\rho_{1,2} = z \pm \sqrt{z^2-1}$$

$$F(\rho,z) = \sum_{n=0}^{\infty}\frac{P_n(z)}{\rho^{n+1}} = \frac{1}{2\pi i}\int_{C_1} d\rho\, F(\rho,z)\,\rho^n$$

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$$= \frac{1}{2\pi i}\int_{C_1} d\rho\,\frac{1}{\rho^{n+1}\sqrt{1-2\rho z+\rho^2}} \quad \rho = z+\sqrt{z^2-1}\cos\phi$$

$$\frac{1}{\pi}\int^{\pi}\frac{d\phi}{(z+\sqrt{z^2-1}\ldots)^n}$$