Assignment 3

Integral Transforms

1. Derive the expressions for the Laplace transforms given in Table 4-1 on p. 119.

2. Using the Laplace transform, derive the final amplitude of a simple harmonic oscillator, initially at rest, subject to an external force

$$F = \begin{array}{ll} F_0 t/\tau & ,\ 0 < t < \tau \\ 0 & ,\ t > \tau \end{array}$$

3. Find the displacement $u\left(x,t\right)$ in an elastic string

$$a^2 u_{xx} = u_{tt},\ 0 < x < l,\ t > 0$$

   that is fixed at its ends

$$u\left(0,t\right) = u\left(l,t\right) = 0,\ t > 0$$

   and is set in motion by plucking it at its center

$$u_t\left(x,0\right) = 0,\ u\left(x,0\right) = \begin{array}{ll} Ax & ,\ 0 \le x \le l/2 \\ A\left(l-x\right) & ,\ l/2 \le x \le l \end{array}$$

4. Consider a uniform rod of length l with initial temperature given by

$$T\left(x,0\right) = T_0 \sin\frac{\pi x}{l},\ 0 \le x \le l$$

   Assume that both ends of the rod are insulated

$$T_x\left(0,t\right) = T_x\left(l,t\right) = 0,\ t > 0$$

   Find a formal series expansion for the temperature $T\left(x,t\right)$

$$\kappa T_{xx}\left(x,t\right) = T_x\left(t\right),\ 0 < x < l,\ t > 0$$

   What is the steady-state temperature as $t \to \infty$?

5. Find the Fourier series of the function assuming that the function is periodically extended outside the original interval

   (a)

$$f(x) = \begin{matrix} -x \ , & -l \leq x < 0 \\ x \ , & 0 \leq x < l \end{matrix}$$

   (b)

$$f(x) = \sin^2 x, \ -\pi \leq x \leq \pi$$

   (c)

$$f(x) = \begin{matrix} 0 \ , & -1 \leq x < 0 \\ x^2 \ , & 0 \leq x < 1 \end{matrix}$$

   (d)

$$\begin{aligned} f(x) &= x, \ -l < x < l \\ f(0) &= f(l) = 0 \end{aligned}$$

6. Find the formal solution of the initial value problem

$$\frac{d^2x}{dt^2} + \omega^2 x = \sum_{n=1} b_n \sin nt$$

$$x(0) = 0, \ \frac{dx}{dt}(0) = 0$$

   if $\omega > 0$ is not equal to a positive integer. How is the solution altered if $\omega = m$, where m is a positive integer?