Assignment 1

Ordinary Differential Equations

1. Find the solution of the initial value problem

$$y' = \frac{y \cos x}{1 + 2y^2},\ y(0) = 1$$

2. Solve the differential equation

$$\left(\frac{y}{x} + 6x\right) + (\ln x - 2)\, y' = 0$$

3. Solve the differential equation

$$y' \left(\frac{x}{y} - \sin y\right) + 1 = 0$$

4. Solve the differential equation

$$y^2 + 2xy + y'x^2 = 0$$

5. (Epidemics) Assume that there is a community of n members containing p infected individuals and q uninfected, $p + q = n$. Denote $x = p/n$, $y = q/n$ proportion of sick and well members respectively. For $n \gg 1$ these can be treated as continuous variables (why?). The rate dx/dt at which the disease spreads is proportional to xy, the number of contacts, with the coefficient of proportionality β. If the initial proportion of the infected population was $x(t = 0) = x_0$, what will it be at a later time t? Investigate your answer both for short time scales, $t \ll \beta^{-1}$, and for $t \to \infty$ and interpret your results.

6. An object of mass m is dropped from rest in a medium that offers resistance proportional to $|v|$, the magnitude of the instantaneous velocity of the object, with the coefficient of proportionality k. Assuming the gravitational force to be constant, find the position of the object at any time.

7. Consider a space vehicle of initial mass m_0 projected upward from the surface of the earth with an initial velocity v_0. Assume that a part of the mass $m_f < m_0$ consists of fuel and is consumed at a constant rate β during the interval $0 \leq t \leq t_1$, t_1 being the time at which the fuel is exhausted. Assume that the exhaust speed $s = |u| = -u$ is also constant. Neglect air resistance and assume that the gravitational force is constant. Find an expression for velocity of the vehicle as a function of time t for $0 \leq t \leq t_1$.

8. Show that $y = x$ is a solution of the Legendre equation of order one

$$\left(1 - x^2\right) y'' - 2xy' + 2y = 0, \ -1 < x < 1$$

and find a second linearly independent solution.

9. Solve the differential equation

$$y'' + 4y = x \exp x + x \sin 2x$$

10. Solve the differential equation

$$y'' + y = \sec x, \ 0 < x < \pi/2$$

11. Solve the differential equation

$$x^2 y'' + 5xy' + 4y = 0$$