2D Green's fn. using Sturm-Liouville (compare pp 271-273)

$$(\nabla^2 + k^2)\, G(r, r'; \theta) = \frac{1}{r}\,\delta(r-r')\,\delta(\theta), \quad -\pi \le \theta \le \pi$$

$$\delta(\theta) = \frac{1}{\pi}\sum_m \frac{1}{\varepsilon_m}\cos m\theta$$

$$G(r, r'; \theta) = \sum_m G_m(r, r')\cos m\theta$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial G_m}{\partial r}\right) - \frac{m^2}{r^2}G_m + k^2 G_m = \frac{1}{r}\delta(r-r')\frac{1}{\pi\varepsilon_m}$$

$$\frac{\partial}{\partial r}\left(r\frac{\partial G_m}{\partial r}\right) + \left(k^2 r - \frac{m^2}{r}\right)G_m = \delta(r-r')\frac{1}{\pi\varepsilon_m}$$

$$p = -r, \quad q = k^2 r - \frac{m^2}{r}$$

$$v_1 = J_m(kr), \quad v_2 = J_m(kr)Y_m(kR) - J_m(kR)Y_m(kr)$$

$$w = \begin{vmatrix} J_m(kr) & J_m(kr)Y_m(kR) - J_m(kR)Y_m(kr) \\ kJ_m'(kr) & k\left(J_m'(kr)Y_m(kR) - J_m(kR)Y_m'(kr)\right) \end{vmatrix}$$

$$= -k\left(J_m(kr)Y_m'(kr) - J_m'(kr)Y_m(kr)\right)J_m(kR)$$

$$= -\frac{2}{\pi r}J_m(kR)$$

$$G_m(r, r') = -\frac{1}{2\varepsilon_m J_m(kR)}\begin{cases} J_m(kr)\left[J_m(kr')Y_m(kR) - J_m(kR)Y_m(kr')\right], & r < r' \\ J_m(kr')\left[J_m(kr)Y_m(kR) - J_m(kR)Y_m(kr)\right], & r > r' \end{cases}$$