Assignment 2

Integration

1. What are the residues of the following functions at the points indicated

   (a)

   $$\frac{\exp az}{z^5} \text{ at } z = 0$$

   (b)

   $$\frac{1}{\sin^3 z} \text{ at } z = 0$$

2. Calculate

   $$\lim_{\epsilon \to 0} \int_{-\infty}^{\infty} \frac{dk}{(k^2 - a^2 - i\epsilon)^3}, \text{ with } a > 0$$

3. Evaluate

   $$\int_0^{\infty} \frac{x^3 dx}{\exp x - 1}$$

4. Evaluate

   $$\int_{-\infty}^{\infty} \frac{dx}{\cosh^3 x}$$

5. Calculate

   $$\int_0^{2\pi} \frac{d\phi}{\alpha + \cos\phi}$$

   (a) When $\alpha > 1$

   (b) When $\alpha = \alpha_0 + i\epsilon$, α_0, ϵ real, $\epsilon > 0$ and $0 < \alpha_0 < 1$, as $\epsilon \to 0$

   (c) When $\alpha = -1$

6. Evaluate

$$\int_0^{2\pi} d\phi \frac{b + a\cos\phi}{a^2 + b^2 + 2ab\cos\phi}, \ |a| \neq |b|$$

7. Evaluate

$$\int_{-\infty}^{\infty} dt \frac{\sin t}{t} \exp ipt$$

8. Evaluate

$$\int_0^{\infty} dx \frac{x^a}{(x+1)^2}, \ -1 < a < 1$$

9. Calculate

$$\int_0^{\infty} dt \cos t^2$$

10. Calculate

$$\int_0^{\infty} \frac{dx}{1 + x^n}$$

11. Calculate

$$\int_0^{\infty} dx \frac{x^2}{x^4 - 2x^2\cos 2\theta + 1}$$