MathPhys - Winter 2003
Quiz 4

1. (10 points) Find the Green's function of the 3D screened Laplace operator

$$\left(\nabla^2 - \alpha^2\right) G_3(\mathbf{r}) = \delta(\mathbf{r})$$

Solution

$$-\left(k^2 + \alpha^2\right) G_3(\mathbf{k}) = \frac{1}{(2\pi)^3}$$

and

$$\begin{aligned} G_3(\mathbf{r}) &= -\frac{1}{(2\pi)^3} \int d^3\mathbf{k} \frac{\exp(i\mathbf{k}\cdot\mathbf{r})}{(k^2+\alpha^2)} = -\frac{1}{2\pi^2 r} \int_0^\infty dk \frac{k \sin kr}{(k^2+\alpha^2)} = -\frac{1}{2i(2\pi)^2 r} \int_{-\infty}^\infty dk \frac{k\left[\exp(ikr) - \exp(-ikr)\right]}{(k^2+\alpha^2)} \\ &= -\frac{2\pi i}{2i(2\pi)^2 r} \left\{ \frac{i\alpha \exp(-\alpha r)}{2i\alpha} - \left[\frac{-i\alpha\left[-\exp(-\alpha r)\right]}{-2i\alpha}\right] \right\} = -\frac{\exp(-\alpha r)}{4\pi r} \end{aligned}$$

2. (10 points) Find the retarded Green's function of the 1D Schrödinger operator

$$\left(i\hbar\frac{\partial}{\partial t} + \frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\right) G_1^R(x,t) = \delta(x,t)$$

Solution

$$\left(i\hbar\frac{\partial}{\partial t} - \frac{\hbar^2}{2m}k^2\right) G_1^R(k,t) = \frac{\delta(t)}{2\pi}$$

whereof

$$G_1^R(k,t) = \frac{\theta(t)}{2\pi i\hbar} \exp\left(-\frac{i\hbar}{2m}k^2 t\right)$$

and

$$\begin{aligned} G_1^R(x,t) &= \frac{\theta(t)}{2\pi i\hbar} \int_{-\infty}^\infty \exp\left(-\frac{i\hbar}{2m}k^2 t + ikx\right) dk = \frac{\theta(t)\sqrt{\pi}}{2\pi i\hbar\sqrt{i\hbar t/2m}} \exp\left(-\frac{x^2}{4i\hbar t/2m}\right) \\ &= -\theta(t)\frac{1+i}{\sqrt{2}\hbar}\sqrt{\frac{m}{2\pi\hbar t}} \exp\left(\frac{im}{2\hbar t}x^2\right) \end{aligned}$$