MathPhys - Fall 2002
Quiz 2

1. (8 points) Evaluate the series

$$\sum_{k=1}^{\infty} \frac{1}{2^k k}$$

Solution

$$S(x) = \sum_{k=1}^{\infty} \frac{x^k}{2^k k}$$

$$S'(x) = \frac{1}{2} \sum_{k=1}^{\infty} \frac{x^{k-1}}{2^{k-1}} = \frac{1}{2} \frac{1}{1 - x/2} = \frac{1}{2 - x}$$

$$S(x) = -\ln(2 - x) + C$$

$$S(0) = 0 = -\ln 2 + C$$

$$S(x) = -\ln(2 - x) + \ln 2$$

$$\sum_{k=1}^{\infty} \frac{1}{2^k k} = S(1) = \ln 2$$

2. (7 points) Evaluate the series

$$\sum_{k=0}^{\infty} \frac{(-1)^k}{n^{2k}}$$

assuming $n > 1$. What happens when $n = 1$?
Solution

$$\sum_{k=0}^{\infty} \frac{(-1)^k}{n^{2k}} = \sum_{k=0}^{\infty} (-1)^k \left(n^{-2}\right)^k = \sum_{k=0}^{\infty} \left(-n^{-2}\right)^k$$
$$= \frac{1}{1 + n^{-2}} = \frac{n^2}{1 + n^2}$$

In the limit of $n = 1$ this gives $1/2$. In reality, the sum $\sum_{k=0}^{N} (-1)^k$ takes on values $1, 0, 1, 0...$ as $N = 0, 1, 2, 3...$