15-Phys-202

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Final

Name ______________________

1. An oscillator consists of a block attached to a spring ($k = 400$ N/m). At some point t, the position (measured form the system's equilibrium location), velocity, and acceleration of the block are $x = 0.100$ m, $v = -13.6$ m/s, and $a = -123$ m/s^2. Calculate

   (a) the frequency of the oscillation,

   (b) the mass of the block,

   (c) the amplitude of the motion.

   Solution

$$\omega = \sqrt{\frac{-a}{x}} = 35.07 \text{ rad/s}$$
$$f = \frac{\omega}{2\pi} = 5.58 \text{ Hz}$$

$$\omega = \sqrt{\frac{k}{m}} \Rightarrow m = \frac{k}{\omega^2} = 0.325 \text{ kg}$$

$$x_m = \sqrt{x^2 + \frac{v^2}{\omega^2}} = 0.400 \text{ cm}$$

2. Two plastic rods curved into semicircles of the same radius R, one of charge $+q$ and the other of charge $-q$, are joined to form a circle of radius R in an xy plane. The x-axis passes through their connecting points with the positively charged rod being in the $y \geq 0$ half-plane, and the charge is distributed uniformly on both rods. What are the magnitude and direction of the electric field $\overrightarrow{E}$ produced at the center of the circle?

   Solution

   From symmetry considerations the net field at the center is twice that of the upper semicircle.

$$
\begin{aligned}
\overrightarrow{E}_{net} &= 2\left(-\widehat{j}\right)\int_{-\pi/2}^{\pi/2}\frac{\lambda\left(Rd\phi\right)\cos\phi}{4\pi\varepsilon_0 R^2} = 2\left(-\widehat{j}\right)\frac{\lambda}{4\pi\varepsilon_0 R}\left[\sin\phi\right]_{-\pi/2}^{\pi/2} \\
&= -\frac{\lambda}{\pi\varepsilon_0 R}\widehat{j} = -\frac{q}{\varepsilon_0\pi^2 R^2}\widehat{j}
\end{aligned}
$$

   where

$$
\lambda = \frac{q}{\pi R}
$$

   is the linear charge density.

3. A point charge $+q$ is at a distance $d/2$ directly above the center of a square of side d. What is the magnitude if the electric flux through the square. *Hint*: Think of a square as one side of a cube of edge d and evaluate the total amount of flux through the cube's surface.

   Solution

   The flux through each face of the cube will be the same for a charge in the center of the cube, so

$$\Phi = \frac{\Phi_{net}}{6} = \frac{q}{6\varepsilon_0}$$

4. An infinite non-conducting sheet has a surface charge density $\sigma = 0.20\ \mu\text{C/m}^2$ on one side. How far apart are the equipotential surfaces whose potentials differ by 25 V? *Hint*: $\Delta V = E\Delta x$.

   Solution

$$\Delta x = \frac{\Delta V}{E} = \frac{2\varepsilon_0 \Delta V}{\sigma} = 2.2 \times 10^{-3}\ \text{m}$$

   where

$$E = \frac{\sigma}{2\varepsilon_0}$$

   is the electric field of an infinite non-conducting sheet.

5. In the figure, battery B supplies 12 V. Find the charge on each capacitor

   (a) first when only switch S_1 is closed

   (b) and later when switch S_2 is also closed.

   Take $C_1 = 1.0\ \mu$F, $C_2 = 2.0\ \mu$F, $C_3 = 3.0\ \mu$F, $C_4 = 4.0\ \mu$F.

   Solution

   When switch S_1 is closed, capacitors 1 and 3 are in series and capacitors 2 and 4 are in series. Consequently,

$$q_1 = q_3 = \frac{C_1 C_3}{C_1 + C_3} V = 9.0\ \mu\text{C}$$

$$q_2 = q_4 = \frac{C_2 C_4}{C_2 + C_4} V = 16\ \mu\text{C}$$

   When switch S_2 is also closed, capacitors 1 and 2 are in parallel and so are 3 and 4. The equivalent capacitance of these pairs are

$$C' = C_1 + C_2 \text{ and } C'' = C_3 + C_4$$

   respectively. C' and C'' are in series, each carrying a charge of

$$q' = q'' = V \frac{(C_1 + C_2)(C_3 + C_4)}{C_1 + C_2 + C_3 + C_4}$$

   which is split between the capacitors1 and 2 and capacitors 3 and 4 in such a way that

$$q' = q_1 + q_2, \ \frac{q_1}{C_1} = \frac{q_2}{C_2} \text{ and } q' = q_3 + q_4, \ \frac{q_3}{C_3} = \frac{q_4}{C_4}$$

   respectively. Consequently we find

$$\begin{aligned} q_i &= V C_i \frac{(C_3 + C_4)}{(C_1 + C_2 + C_3 + C_4)}, \ i = 1, 2 \\ q_i &= V C_i \frac{(C_1 + C_2)}{(C_1 + C_2 + C_3 + C_4)}, \ i = 3, 4 \end{aligned}$$

   so that

$$\begin{aligned} q_1 &= 8.4\ \mu\text{C}, \ q_2 = 17\ \mu\text{C} \\ q_3 &= 11\ \mu\text{C}, \ q_4 = 14\ \mu\text{C} \end{aligned}$$

6. In the figure,

   (a) what is the equivalent resistance of the network shown?

   (b) what is the current in each resistor?

   Put $R_1 = 60\ \Omega$, $R_2 = R_3 = 100\ \Omega$, $R_4 = 200\ \Omega$, and $\mathcal{E} = 10$ V; assume the battery is ideal.

   Solution

   R_2, R_3, and R_4 are connected in parallel whose equivalent resistance is

$$R = \frac{R_2 R_3 R_4}{R_2 R_3 + R_2 R_4 + R_3 R_4} = 40\ \Omega$$

   This resistor is in series with resistor R_1, so that the equivalent of the network is

$$R_{eq} = R + R_1 = 100\ \Omega$$

   The currents are found as follows ($i_2 = i_3 = 2i_4$, $i_1 = i_2 + i_3 + i_4 = 5i_2/2$)

$$\begin{aligned} i_1 &= \frac{\mathcal{E}}{R_{eq}} = 0.1\ \text{A} \\ i_2 &= i_3 = 0.04\ \text{A},\ i_4 = 0.02\ \text{A} \end{aligned}$$

7. An alpha particle ($q = +2e$, $m = 4.00$ u) travels in a circular path of radius 9.00 cm in a uniform magnetic field with $B = 1.20$ T. Calculate

   (a) its speed

   (b) its period of revolution

   (c) its kinetic energy in electron-volts

   (d) the potential difference through which it would have to be accelerated to achieve this energy.

   Solution

$$v = \frac{r\,(2e)\,B}{m_\alpha} = 5.20 \times 10^6 \text{ m/s}$$

$$T = \frac{2\pi r}{v} = 1.09 \times 10^{-7} \text{ s}$$

$$K = \frac{m_\alpha v^2}{2} = 5.60 \times 10^5 \text{ eV}$$

$$V = \frac{K}{(2e)} = 2.80 \times 10^5 \text{ V}$$

8. Two long, straight, parallel wires, separated by 0.75 cm, are perpendicular to the plane of the page as shown in the figure. Wire I carries a current of 6.0 A into the page. What must be the current (magnitude and direction) in wire 2 for the resultant magnetic field at point P to be zero?

   Solution

   The current in wire 2 must be out of page. Using

$$B = \frac{\mu_0 i}{2\pi r}$$

   and the condition

$$B_{P1} = B_{P2}$$

   we find

$$i_2 = i_1 \frac{r_2}{r_1} = 4.0 \text{ A}$$

9. A circular coil has a 10.0 cm radius and consists of 30.0 closely wound turns of wire. An externally induced magnetic field of 2.60 mT is perpendicular to the coil.

   (a) If no current is in the coil, what magnetic flux links its turns?

   (b) When the current in the coil is 3.80 A in a certain direction, the net flux through the coil is found to vanish. What is the inductance of the coil?

   Solution

   Flux linkage is

$$N\Phi_B = NBA = NB\left(\pi r^2\right) = 2.45 \times 10^{-3} \text{ W}$$

   The inductance of the coil for which the flux vanishes is

$$L = \frac{N\Phi_B}{i} = 6.45 \times 10^{-5} \text{ H}$$