15-Phys-202

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Exam 2

Name ____________________

Useful formulae and constants:

$$\varepsilon_0 = 8.85 \times 10^{-12}\ \mathrm{C^2/N \cdot m^2}$$

$$k = \frac{1}{4\pi\varepsilon_0} = 8.99 \times 10^{9}\ \mathrm{N \cdot m^2/C^2}$$

$$\Phi = \frac{q_{enclosed}}{\varepsilon_0}$$

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

$$\int \frac{x dx}{\sqrt{x^2 + a^2}} = \sqrt{x^2 + a^2}$$

$$q = CV$$

$$C = \frac{\varepsilon_0 \kappa A}{d}$$

$$E = \frac{V}{d}$$

1. A spherical conducting shell with a net charge of $-3.00Q$ contains a particle of charge $+5.00Q$ in the hollow. The shell has an inner radius of 0.80 m and an outer radius of 1.4 m. For $Q = 1.0$ nC, what are the magnitudes and the directions of the electric field at

   (a) point A at radius 0.500 m,

   (b) point B at radius 1.00 m, and

   (c) point C at radius 2.00 m?

   Solution

   At A, the only field contributing is from the $+5.00Q$ particle in the hollow and the field is

$$E = k\frac{(5Q)}{(0.5)^2} = 20kQ = 20 \times 8.99 \times 10^9 \times 10^{-9} = 180 \text{ V/m}$$

   directed radially outward.

   At B, the field is zero since it is inside the conducting material.

   At C, the field is

$$E = k\frac{(5Q - 3Q)}{(2)^2} = \frac{kQ}{2} = \frac{8.99 \times 10^9 \times 10^{-9}}{2} = 4.5 \text{ V/m}$$

   and is directed radially outward.

2. A plastic disk has a radius $R = 2.20$ cm. Its surface charge density is 1.50×10^{-6} C/m^2 from $r = 0$ to $R/2$ and 8.00×10^{-7} C/m^2 from $r = R/2$ to R.

   (a) What is the total charge of the disk?

   (b) With $V = 0$ at infinity, what is the electric potential at a point on the central axis of the disk, at a distance $z = R/2$ from the center of the disk?

Solution

The total charge of the disk is

$$\begin{aligned} q &= \int_0^{R/2} 2\pi\sigma_1 r dr + \int_{R/2}^{R} 2\pi\sigma_2 r dr = 2\pi\left[\sigma_1 \frac{1}{2}\left(\frac{R}{2}\right)^2 + \sigma_2 \frac{1}{2}\left(R^2 - \left(\frac{R}{2}\right)^2\right)\right] \\ &= \frac{\pi}{4}R^2\left[\sigma_1 + 3\sigma_2\right] = \frac{\pi}{4}\left(2.20 \times 10^{-2}\right)^2\left[1.50 \times 10^{-6} + 3 \times 8.00 \times 10^{-7}\right] = 1.48 \text{ nC} \end{aligned}$$

The electric potential is

$$\begin{aligned} V &= k\int_0^{R/2} \frac{\sigma_1 (2\pi r)\, dr}{\sqrt{r^2 + z^2}} + \int_{R/2}^{R} \frac{\sigma_2 (2\pi r)\, dr}{\sqrt{r^2 + z^2}} = \\ & 2\pi k\left[\sigma_1\left(\sqrt{\left(\frac{R}{2}\right)^2 + z^2} - z\right) + \sigma_2\left(\sqrt{R^2 + z^2} - \sqrt{\left(\frac{R}{2}\right)^2 + z^2}\right)\right] \\ &\overset{z=R/2}{=} \frac{1}{2\varepsilon_0}\left[\sigma_1\left(\sqrt{\left(\frac{R}{2}\right)^2 + \left(\frac{R}{2}\right)^2} - \frac{R}{2}\right) + \sigma_2\left(\sqrt{R^2 + \left(\frac{R}{2}\right)^2} - \sqrt{\left(\frac{R}{2}\right)^2 + \left(\frac{R}{2}\right)^2}\right)\right] \\ &= \frac{R}{4\varepsilon_0}\left[\left(\sqrt{2} - 1\right)\sigma_1 + \sigma_2\left(\sqrt{5} - \sqrt{2}\right)\right] \\ &= \frac{\left(2.20 \times 10^{-2}\right)}{4\left(8.85 \times 10^{-12}\right)}\left[\left[\left(\sqrt{2} - 1\right)\left(1.50 \times 10^{-6}\right) + \left(\sqrt{5} - \sqrt{2}\right)\left(8.00 \times 10^{-7}\right)\right]\right] = 795 \text{ V} \end{aligned}$$

3. Two sheets of aluminum foil have the same area, a separation of 1.0 mm and (together) a capacitance of 10 pF, and they are charged to 12 V.

   (a) Calculate the area of each sheet.

   The separation is now decreased by 0.10 mm with the charge held constant.

   (b) What is the new capacitance?

   (c) By how much does the potential difference change?

   Solution

   The area of each sheet is

$$A = \frac{Cd}{\varepsilon_0} = 1.1 \times 10^{-3}\ \mathrm{m}^2$$

   After the distance decreases to $d' = 0.9$ mm,

$$C' = C\frac{d}{d'} = 10\frac{1.0}{0.9} = 11\ \mathrm{pF}$$

   and

$$V' - V = \frac{q}{C'} - V = V\left(\frac{d'}{d} - 1\right) = -1.2\ \mathrm{V}$$

4. Two parallel-plate capacitors A and B are connected in parallel across a 600 V battery. Each plate has area 80.0 cm^2; the plate separation is 3.00 mm. Capacitor A is filled with air; capacitor B is filled with a dielectric of dielectric constant $\kappa = 2.60$. Find the magnitude of the electric field within the dielectric of capacitor B and the air of capacitor A.

   Solution

   Since the capacitors are in parallel, they have the same voltage. They also have the same distance between the plates. Therefore the field is the same and is given by

$$E = \frac{V}{d} = \frac{600 \text{ V}}{0.003 \text{ m}} = 2.00 \times 10^5 \text{ V/m}$$

   (Note: the charge on the plates of the dielectric-filled capacitor is larger than that of the air-filled capacitor, $q_B = \kappa\varepsilon_0 A > q_A = \varepsilon_0 A$).