15-Phys-202

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Exam 1

Name _____

Useful formulae and constants:

$$x = x_m \cos(\omega t + \phi)$$

$$F = k\frac{|q_1|\,|q_2|}{r^2},\ k = \frac{1}{4\pi\varepsilon_0} = 8.99 \times 10^9\ \mathrm{N \cdot m^2/C^2}$$

$$e = 1.60 \times 10^{-19}\ \mathrm{C}$$

1. A particle undergoes simple harmonic motion along an x axis about $x = 0$ with a period of 0.40 s and an amplitude of 0.10 m. At $t = 0$ the particle is at its position of maximum negative displacement – that is, at $x = -0.10$ m. Write expressions, as functions of time, for

   (a) the particle's position

   and

   (b) its velocity.

   Solution

   At $t = 0$

$$x = x_m \cos\phi = -x_m$$

   whereof

$$\cos\phi = -1$$

   and

$$\phi = \pi$$

   Also,

$$\omega = \frac{2\pi}{T} = 5\pi \text{ rad/s}$$

   Then

$$x = 0.10\cos(5\pi t + \pi) = -0.10\cos 5\pi t$$

   and

$$v = \frac{dx}{dt} = -0.50\pi\sin(5\pi t + \pi) = 0.50\pi\sin 5\pi t$$

   where t is in seconds and x is in meters.

2. Three identical conducting spheres A, B, and C form an equilateral triangle of side length d and have initial charges of $-2Q$, $-4Q$, and $8Q$ respectively.

   (a) What is the magnitude of the electric force between spheres A and C?

   The following steps are then taken: A and B are connected by a thin wire and then disconnected; B is grounded by the wire and the wire is then removed; B and C are connected by the wire and then disconnected. What now are the magnitudes of the electrostatic force

   (b) between spheres A and C?

   (c) between spheres B and C?

   Hint: When grounded, a conducting object loses its charge and becomes neutral.

   Solution

   Initially, $q_A = -2Q$, $q_B = -4Q$, $q_C = 8Q$ and

$$F_{AC} = \frac{|-2Q|\,|8Q|}{4\pi\varepsilon_0 d^2} = \frac{4Q^2}{\pi\varepsilon_0 d^2}$$

   After making contact with each other, both A and B have a charge of

$$\frac{-2Q - 4Q}{2} = -3Q = Q_A$$

   When B is grounded, its charge is zero. After making a contact with C, both B and C have a charge of

$$\frac{0 + 8Q}{2} = 4Q = Q_B = Q_C$$

   where Q_A, Q_B, and Q_C denote the final charges. Therefore,

$$F_{AC} = \frac{|-3Q|\,|4Q|}{4\pi\varepsilon_0 d^2} = \frac{3Q^2}{\pi\varepsilon_0 d^2}$$

$$F_{BC} = \frac{|4Q|\,|4Q|}{4\pi\varepsilon_0 d^2} = \frac{4Q^2}{\pi\varepsilon_0 d^2}$$

3. An electron is placed at each corner of an equilateral triangle having sides 20 cm long. What is the magnitude of the electric field at the midpoint of one of the sides?

   Solution

   The contributions to the net field from the two electrons at the nearest corners cancel, so that we only need to compute the contribution from the electron at the far corner at a distance

$$a = \sqrt{(0.2)^2 - (0.1)^2}\ \text{m}$$

   Therefore,

$$E = k\frac{e}{a^2} = 8.99 \times 10^9 \frac{1.60 \times 10^{-19}}{0.03}\ \text{N/C} = 4.8 \times 10^{-8}\ \text{N/C}$$